Some lines of research in Sting theory in Mathematics and Physics

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Outline

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Introduction: String Theory

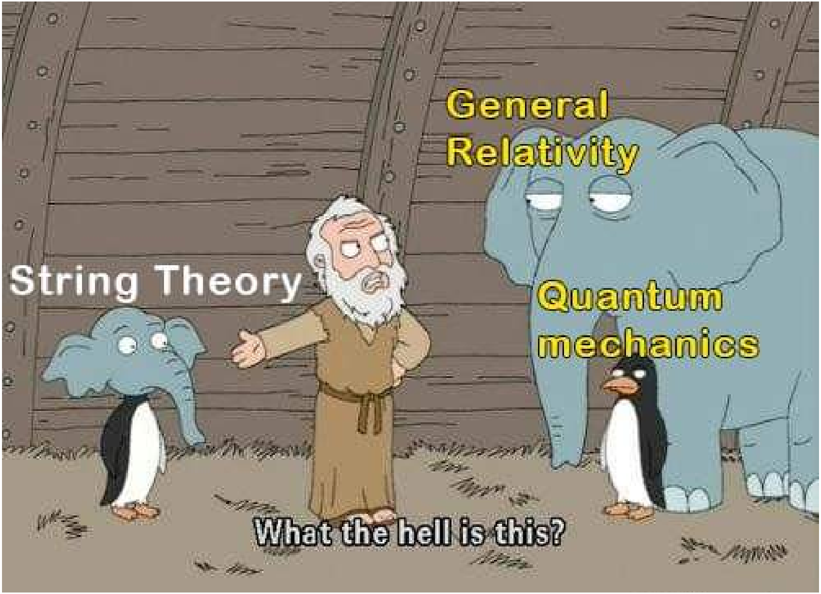
Why study string theory?

- It is a theory of quantum gravity.
- It unifies all of the forces of nature in one theory.
- It can explain the cosmological constant.
- Why not?

But is it Physics?

- Hasn't made a testable prediction.
- Why do we need quantum gravity anyway?

There is some scepticism in certain parts of the physics community:



Introduction: Quantum Gravity

The Einstein-Hilbert action for General Relativity is

$$S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R \tag{1}$$

We may write Newtons constant in terms of the Planck mass as

$$8\pi G_N = \frac{\hbar c}{M_{pl}^2} \tag{2}$$

where $M_{pl} \approx 2 \times 10^{18} Gev$. If we take perturtbations around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{pl}} h_{\mu\nu} \tag{3}$$

we obtain schematically

$$S_{EH} = \int d^4x (\partial h)^2 + \frac{1}{M_{pl}} h(\partial h)^2 + \frac{1}{(M_{pl})^2} h^2 (\partial h)^2 + \cdots \tag{4}$$

a standard looking QFT with an infinite series of interaction terms.

Introduction: Quantum Gravity

- The interactions are surpressed by powers of M_{pl}.
- The quantum perturbation theory expansion is therefore an expansion in E^2/M_{pl}^2 where E is the energy of the process of interest.
- Unfortunately it is non-renormalizable - it requires an infinite number of counter terms to make the theory finite as loop order increases.
- Other approaches - Loop quantum gravity (change the quatization proceedure), non-commutative geometry (change the description of space-time).

Introduction: Supergravity

Supergravity:

- Low energy limit of string theory
- Effective theory - many strings calculations done in sugra limit.

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- SUSY broken in hidden gravitational sector
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but

- Essentially ruled out by a Higgs at 125Gev
- Neutralino dark matter abundance, Susy particle mass bounds.

String Theory and Phenomonology

Many problems!

- 4D - Strings live in 10D and Membranes in 11D
- Sugra is either asymptotically flat or Anti-de Sitter, but we live in a de Sitter universe.
- Large number of possible vacua, the "Landscape". Predictability?
- Supersymmetry - If it exists in nature it is broken at at energies currently accessible by colliders.

4D

Compactifications - String phenomonology:

- String theory is considered on a suitable background with 4 non-compact dimensions.
- The compactification manifold is restricted by the amount of residual supersymmetry in 4D.
- The details on the compactification manifold give the details of the 4D QFT.

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Brane worlds:

- Known physics restricted to 4D manifold in higher dimensional space time
- extra dimensions may be large
- branes always break at least half of the supersymmetry.
- Difficult to produce supersymmetric brane worlds without singularities.

dS

- It is impossible to produce a supersymmetric dS theory of gravity.
- de Sitter SUGRA proposed a few years ago in which SUSY is spontaneously broken.
- Compactification origin?
- related to world volume theory for $\overline{D3}$ branes.

Landscape

Too many vacua: Related to

- Compactification mechanism - what is it?.
- Strings live in 10D but "grows" a dimension in the strong coupling limit to become M-theory.
- 1st quantized theory: Change to string field theory and higher spin theory.

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- Duality invariant formulation:
- T-duality invariant doubled theory.
- U duality invariant formulation from E10/11
- Is there some principle or dymanics that select the compactification manifold away from stringy energies?

Susy Breaking

- Related to all the others.
- In particular how many supersymmetries are broken dynamically and how many by the brane/compactification?
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- LHC (or beyond) finds low energy superssymmetry and we can fit the breaking mechanism with confidence.
- A classification of supersymmetric backgrounds tells us the compactification manifold.
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Overly pessimistic:

- No low energy supersymmetry
- landscape problem remains
- compactification manifold only resticted by not being supersymmetric and that the resulting gauge group contains that of the standard model.

- Holography to the rescue - Our work wasn't pointless!
- Learn about strong coupling limit of field theories, susy provides calculability on both sides of the correspondence to learn more about it.
- Description of non-perturbative aspects of CFTs in flat space. Holographic QCD?
- We need that rather than having an infinite number of colors, and being at infinite coupling some method to move away from these limits. String theory effects in supergravity provide an expansion away from these regimes in the inverse coupling and inverse of the number of colours.
- Doing string theory properly should lead to new methods in QFT including in CMT.

Some research lines in Strings and related areas

- Non-perturbative description: String field theory, doubled and exceptional field theory (generalized complex and exceptional geometry, Kac Moody Algebras)
- Field theory: Conformal fields theories on different geometric backgrounds, classification, applications to HEP and CMT. (Algebras, Kac-moody algebras, representation theory, category theory)
- The landscape and swampland - Quantum consistency of theories in lower dimensions and their possible vacua. (Novel methods in QFT, effective field theory)
- String cosmology - Very (very very) early universe, when size of universe was small. Source for inflation.
- String phenomoenology - Field theory from string theory on compactification manifolds. (algebraic geometry, topology, differential geometry, index theory)
- Non-commutative geometry. (C^* algebras, algebraic K-theory, category theory)
- Many others and.......

Susy breaking

- SUSY and its breaking is key to connect string theory with phenomenology
- A classification of susy backgrounds and branes is necessary if we are to be able to make such a connection.
- How do we achieve that?

We use tools from the areas in mathematics:

- Group, algebra and representation theory.
- Differential and spin geometry, Clifford algebras.
- Algebraic geometry, topology, index theorems.
- Differential equations,

and we apply them to

- Small black holes.
- The very early universe.
- Field theory in flat space via holography.

Introduction to susy solutions of off-shell sugras

- Supersymmetric solutions of a particular supergravity have to solve two sets of equations:
  - Killing spinor equations: "Supersymmetric"
  - Equations of motion of the theory: "Solution"

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- Supersymmetric solutions of a particular supergravity have to solve two sets of equations:
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  - Equations of motion of the theory: "Solution"

- Killing spinor identities relate components of equations of motion to each other for supersymmetric configurations in supergravity theories
- Tell us which equations of motion are automatically solved by the supersymmetric geometries
- If the supersymmetry in a theory is realised off-shell then we don't even need to know the action, i.e. the specific theory under consideration.
- Can prove all orders results in effective supergravity description of string theory.
- Apply equally to any higher derivative supergravity.

Introduction to susy solutions of off-shell sugras

- Normally horrible to try and supersymmetrize higher derivate (string correction or not) actions on-shell, as we need to change the susy transformations and the action.
- Move to off-shell formulation: We will use the superconformal formalism.
  - Superconformal group biggest possible for S-matrix, supersymmetry is realized off-shell.
  - Matter couplings easier to find.
  - Contains superpoincare, so the superpoincare theories can always be obtained by a suitable gauge fixing.
  - Supersymmetric completions of most curvature squared terms are known.

Off-shell Killing spinor identities

- In work of Ortin & Kallosh and Ortin & Bellorin the Killing spinor identities were derived.
- The derivation does not require that the supersymmetric action is known, just that the action is supersymmetric under the given supersymmetry variations of the fields.
- In work of Meessen (2007) the Killing spinor identities were used in the off-shell $\mathcal{N} = 2$ $d = 5$ superconformal theory to show that the maximally supersymmetric vacuua of the two derivative theory are the vacua of arbitrarily higher derivative corrected theories, up to a generalization of the very special geometry condition.
- Here we will be interested in what they have to say about solutions with less supersymmetry.

Off-shell Killing spinor identities

Lets derive the Killing spinor identities. Let $S[\phi_b, \phi_f]$ be any supergravity action, constructed in terms of bosonic fields ϕ_b and fermionic fields ϕ_f. Let us further assume $S[\phi_b, \phi_f]$ is the spacetime integral of a Lagrangian density:

$$S[\phi_b, \phi_f] = \int d^d x \sqrt{g} \mathcal{L}[\phi_b, \phi_f] \; .$$

The invariance under supersymmetry transformations of the action can be written

$$\delta_Q S[\phi_b, \phi_f] = \int d^d x \sqrt{g} \, \{ \mathcal{L}_b[\phi_b, \phi_f] \delta_Q \phi_b[\phi_b, \phi_f] \\ + \mathcal{L}_f[\phi_b, \phi_f] \delta_Q \phi_f[\phi_b, \phi_f] \} = 0 \; ,$$

where δ_Q denotes a local supersymmetry transformation of arbitrary parameter, subscripts b, f denote functional derivative with respect to ϕ_b, ϕ_f respectively, and a sum over fields is understood.

Off-shell Killing spinor identities

Next consider a second variation of the action functional by varying $\delta_Q S[\phi_b, \phi_f]$ with respect to fermionic fields only. Since $\delta_Q S[\phi_b, \phi_f]$ is identically zero for arbitrary ϕ_b, ϕ_f, we have

$$\delta_Q S[\phi_b, \phi_f + \delta_F \phi_f] = 0$$

and we set the fermions to zero after the variation. Hence we get

$$\begin{aligned} &\delta_F \delta_Q S|_{\phi_f=0} = 0 \\ &= \int d^d x \sqrt{|g|}\, [(\delta_F \mathcal{L}_b)(\delta_Q \phi_b) \\ &\quad + \mathcal{L}_b(\delta_F \delta_Q \phi_b) + (\delta_F \mathcal{L}_f)(\delta_Q \phi_f) + \mathcal{L}_f(\delta_F \delta_Q \phi_f) \Big]_{\phi_f=0} . \end{aligned}$$

Since $\delta_Q \phi_b$ and $\mathcal{L}_f$ are odd in fermions we are left with

$$\int d^d x \sqrt{|g|}\, [(\mathcal{L}_b(\delta_F \delta_Q \phi_b) + (\delta_F \mathcal{L}_f)(\delta_Q \phi_f)]_{\phi_f=0} = 0 .$$

Off-shell Killing spinor identities

Calculating $(\delta_F \mathcal{L}_f)_{\phi_f=0}$ requires knowledge of the entire Lagrangian, not only its bosonic truncation. However if we restrict ourselves to supersymmetry transformations having Killing spinors as parameters, δ_K, we have

$$(\delta_K \phi_f)_{\phi_f=0} = 0 \ .$$

Note that

$$\mathcal{L}_b := \frac{1}{\sqrt{|g|}} \frac{\delta S[\phi_b, \phi_f]}{\delta \phi_b} = \frac{1}{\sqrt{|g|}} \frac{\delta S_B[\phi_b]}{\delta \phi_b} + \frac{1}{\sqrt{|g|}} \frac{\delta S_F[\phi_b, \phi_f]}{\delta \phi_b} \ ,$$

where the last term vanishes if $\phi_f = 0$.

Off-shell Killing spinor identities

We are thus led to define

$$\mathcal{E}_b := \frac{1}{\sqrt{|g|}} \frac{\delta S_B[\phi_b]}{\delta \phi_b} \ ,$$

so that bosonic equations of motion take the form

$$\mathcal{E}_b = 0 \ .$$

Thus the Killing spinor identities may be written as

$$\int d^d x \sqrt{|g|}\ \mathcal{E}_b (\delta_F \delta_K \phi_b)_{\phi_f = 0} = 0 \ .$$

$\mathcal{N} = 2,\ d = 5$ off-shell KSIs

All we need to derive the Ksis for this off-shell are the variation of the fields under supersymmetry. After gauge fixing the superconformal theory to super-Poicare these are

$$
\begin{aligned}
\delta e_\mu^a &= -2i\bar{\epsilon}\gamma^a\psi_\mu \\
\delta v_{ab} &= -\tfrac{1}{8} i\bar{\epsilon}\gamma_{ab}\chi + \cdots \\
\delta D &= -\tfrac{1}{3} i\bar{\epsilon}\gamma^{\mu\nu}\chi v_{\mu\nu} - i\bar{\epsilon}\gamma^\mu\nabla_\mu\chi + \tfrac{4i}{3}\bar{\epsilon}^{\mathbf{i}}\gamma_\mu V^\mu_{\mathbf{ij}}\chi^{\mathbf{j}} + \cdots \\
\delta V_\mu^{\mathbf{ij}} &= -\tfrac{i}{4}\bar{\epsilon}^{(\mathbf{i}}\gamma_\mu\chi^{\mathbf{j})} + \cdots \\
\delta A_\mu^I &= -2i\bar{\epsilon}\gamma_\mu\Omega^I + \cdots \\
\delta M^I &= 2i\bar{\epsilon}\Omega^I \\
\delta Y^{I\mathbf{ij}} &= 2i\bar{\epsilon}^{(\mathbf{i}}\gamma^a\nabla_a\Omega^{\mathbf{j})I} - 2i\bar{\epsilon}^{(\mathbf{i}}\gamma^a V_a{}^{\mathbf{j})}{}_{\mathbf{k}}\Omega^{\mathbf{k}I} - \tfrac{2i}{3} V_a^{\mathbf{k}(\mathbf{i}}\bar{\epsilon}_{\mathbf{k}}\gamma_a\Omega^{\mathbf{j})} \\
&\quad - \tfrac{i}{3}\bar{\epsilon}^{(\mathbf{i}}\gamma_{ab}v^{ab}\Omega^{\mathbf{j})I} - \tfrac{i}{4}\bar{\epsilon}^{(\mathbf{i}}\chi^{\mathbf{j})}M^I\ .
\end{aligned}
$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

- We ignored term involving the gravitino in the variations, apart from in the veilbein variation.
- This is becasue we shall choose to solve the Einstein equation last. - It is usually the most involved in any case.
- In particular, if we assume that when we look to solve the Eintein equation that all other eoms have been solved first, we can ignore the ... terms above.

So if we set

$$\mathcal{E}(e)_a^\mu := \frac{1}{\sqrt{|g|}} \frac{\delta S}{\delta e_\mu^a}$$

we get

$$\mathcal{E}(e)_a^\mu \gamma^a \epsilon^{\mathbf{i}} \Big|_{\text{other bosons on-shell}} = 0 \ .$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

To proceed we will need one more ingredient, the gravitino variation which reads

$$\delta\psi_\mu^{\mathbf{i}} = \left[\nabla_\mu + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}v^{ab}\gamma_\mu\gamma_{ab}\right]\epsilon^{\mathbf{i}} - V_\mu{}^{\mathbf{i}}{}_{\mathbf{j}}\epsilon^{\mathbf{j}} + \tfrac{1}{3}\gamma_\mu\gamma^a V_a^{\mathbf{i}}{}_{\mathbf{j}}\epsilon^{\mathbf{j}} = 0\,.$$

Let us now write the KSI associated to a variation of gauginos. We set

$$\mathcal{E}(A)_I^\mu := \frac{1}{\sqrt{|g|}}\frac{\delta S}{\delta A^I_\mu}$$

$$\mathcal{E}(M)_I := \frac{1}{\sqrt{|g|}}\frac{\delta S}{\delta M^I}\,,$$

$$\mathcal{E}(Y)_{I\mathbf{ij}} := \frac{1}{\sqrt{|g|}}\frac{\delta S}{\delta Y^{I\mathbf{ij}}}\,,$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

We obtain

$$0 = \int d^d x \sqrt{|g|} \left[\mathcal{E}(A)_I^\mu \left(-2i\bar{\epsilon}^{\mathbf{i}} \gamma_\mu \right) + \mathcal{E}(M)_I (2i\bar{\epsilon}^{\mathbf{i}}) + \mathcal{E}(Y)_{I\mathbf{jk}} (2i\bar{\epsilon}^{\mathbf{j}}) \gamma^a V_a^{\mathbf{ki}} \right.$$
$$\left. + \tfrac{2i}{3} \mathcal{E}(Y)^{\mathbf{i}}_{I\mathbf{j}} V_a^{\mathbf{kj}} \bar{\epsilon}_{\mathbf{k}} \gamma_a + \mathcal{E}(Y)_{I\mathbf{j}}{}^{\mathbf{i}} (\tfrac{i}{3} \bar{\epsilon}^{\mathbf{j}} \gamma^{ab} v_{ab}) \right] \delta\Omega^I_{\mathbf{i}}$$
$$+ \mathcal{E}(Y)^{\mathbf{i}}_{I\mathbf{j}} (-2i\bar{\epsilon}^{\mathbf{j}} \gamma^a) \nabla_a \delta\Omega^I_{\mathbf{i}} \; .$$

Integrating by parts and using the fact that the gravitino Killing spinor equation implies

$$\gamma^a \nabla_a \epsilon^{\mathbf{i}} = \tfrac{1}{6} (v \cdot \gamma) \epsilon^{\mathbf{i}} - \tfrac{2}{3} V_{\mathbf{j}}^{a\mathbf{i}} \gamma_a \epsilon^{\mathbf{j}}$$

we obtain

$$\mathcal{E}(A)_I^\mu \gamma_\mu \epsilon^{\mathbf{i}} - \mathcal{E}(M)_I \epsilon^{\mathbf{i}} - \mathcal{E}(Y)_{I\mathbf{jk}} \gamma^a V_a^{\mathbf{ki}} \epsilon^{\mathbf{j}}$$
$$- \mathcal{E}(Y)_I^{\mathbf{ik}} \gamma_a V^a_{\mathbf{jk}} \epsilon^{\mathbf{j}} - \left(\nabla^a \mathcal{E}(Y)_I{}^{\mathbf{i}}{}_{\mathbf{j}} \right) \gamma_a \epsilon^{\mathbf{j}} = 0 \; .$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

Finally we consider the KSI associated with the auxiliary fermion. We define

$$\mathcal{E}(v)^{ab} := \frac{1}{\sqrt{|g|}}\frac{\delta S}{\delta v_{ab}}\mathcal{E}(D) \quad := \frac{1}{\sqrt{|g|}}\frac{\delta S}{\delta D}\mathcal{E}(V)^{\mu}_{\mathbf{ij}} := \frac{1}{\sqrt{|g|}}\frac{\delta S}{V^{\mathbf{ij}}_{\mu}}\ ,$$

and thus obtain

$$\begin{aligned} 0 = \int d^5x\sqrt{|g|}\,\Big[&-\tfrac{i}{8}\mathcal{E}(v)^{ab}\bar{\epsilon}^{\mathbf{i}}\gamma_{ab} + i\mathcal{E}(D)\bar{\epsilon}^{\mathbf{j}}\gamma_a V^{a\mathbf{i}}{}_{\mathbf{j}} - \tfrac{i}{3}\mathcal{E}(D)v^{ab}\bar{\epsilon}^{\mathbf{i}}\gamma_{ab} \\ &-\mathcal{E}(D)\tfrac{4i}{3}\bar{\epsilon}^{\mathbf{j}}V^{\mathbf{i}}_{a\mathbf{j}}\gamma^a - \tfrac{i}{4}\mathcal{E}(V)^{\mu\mathbf{i}}{}_{\mathbf{j}}\bar{\epsilon}^{\mathbf{j}}\gamma_\mu + \tfrac{i}{4}\mathcal{E}(Y)^{\mathbf{i}}_{I\mathbf{j}}\bar{\epsilon}^{\mathbf{j}}M^I\Big]\,\delta\chi_{\mathbf{i}} \\ &+\left[-i\bar{\epsilon}\mathcal{E}(D)\gamma^\mu\right]\nabla_\mu\delta\chi\ . \end{aligned}$$

Integrating the last term by parts, discarding the total derivative and making use of the gravitino Killing spinor equation we obtain

$$\begin{aligned} &\left[\tfrac{1}{8}\mathcal{E}(v)^{ab} + \tfrac{1}{2}\mathcal{E}(D)v^{ab}\right]\gamma_{ab}\epsilon^{\mathbf{i}} + \nabla^a\mathcal{E}(D)\gamma_a\epsilon^{\mathbf{i}} \\ &\qquad + \tfrac{1}{4}\mathcal{E}(V)^{a\mathbf{i}}{}_{\mathbf{j}}\gamma_a\epsilon^{\mathbf{j}} + \tfrac{1}{4}\mathcal{E}(Y)_I{}^{\mathbf{i}}{}_{\mathbf{j}}M^I\epsilon^{\mathbf{j}} - 2\mathcal{E}(D)V^{a\mathbf{i}}{}_{\mathbf{j}}\gamma_a\epsilon^{\mathbf{j}} \quad = \quad 0\ . \end{aligned}$$

Spinorial Geometry and classifying supersymmetric configurations

- Supersymmetric configurations solve the Killing spinor equations, i.e. the vanishing of the supersymmetry variations of the fermions on purely bosnic backgrounds.
- Demanding the vanishing of the gravitino variation for a bosonic background implies

$$\delta\psi^i_\mu = \left[\nabla_\mu + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}v^{ab}\gamma_\mu\gamma_{ab}\right]\epsilon^i = 0 \ .$$

- From the vanishing of the gaugino variation for a bosonic background one has

$$\delta\Omega^{Ii} = \left[-\frac{1}{4}F^I_{ab}\gamma^{ab} - \frac{1}{2}\gamma^\mu\partial_\mu M^I - \frac{1}{3}M^I v^{ab}\gamma_{ab}\right]\epsilon^i = 0 \ .$$

- the vanishing of the auxilary fermion variation for a bosonic background we get

$$\delta\chi^i = \left[D - 2\gamma^c\gamma^{ab}\nabla_a v_{bc} - 2\gamma^a\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(v\cdot\gamma)^2\right]\epsilon^i = 0 \,.$$

Spinorial Geometry and classifying supersymmetric configurations

- In order to solve these equations, in the past the bilineaers methods were used. Form a bilinear out of spinors, then demand that the equations above hold - restricts the form of the spin connection and matter fields.
- Awkward to solve explicitly for the spinors ϵ, extensive use of Fierz identities.
- Gillard Gran & Papadopoulos introduced the spinorial geometry techniques:
  - Use Clifford isomorphim to write the space of spinors in terms of an exterior algebra and the action of the gamma matrices as combination of wedge and exterior products on this space.
  - Choose a particular basis of gamma matrices so they act as creation or anihilation matrices - wedge or interior product.
  - Use the Spin(1,4) gauge freedom in the above equations to write representatives for the spinors - up to local lorentz transformations on the bosonic fields

Spinorial Geometry and classifying supersymmetric configurations

- Now we have explicit representatives for the spinors. Can solve the equations "easily", and more importantly systematically.
- If we leave in all the auxiliary fields the result will be true to all orders generically - but some may not occur due to the imposition of the equations of motion. Eoms imply less general geometry. Supersymmetric configurations are more general than supersymmetric solutions to a given theory (generically).
- Lets see an example.

Weyl squared corrected $\mathcal{N} = 2$, $D = 5$ supergravity coupled to Abelian vector multiplets

As discussed by Castro et al

Lagrangian:

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 \ .$$

where at two derivative level we have

$$\mathcal{L}_2 = \mathcal{L}_V + \mathcal{L}_H = \tfrac{1}{2}D(\mathcal{N} - 1) - \tfrac{1}{4}R(\mathcal{N} + 3) + v^2(3\mathcal{N} + 1)$$
$$+ 2\mathcal{N}_I v^{ab} F^I_{ab} + +\mathcal{N}_{IJ}\left(\tfrac{1}{4}F^I_{ab}F^{Iab} - \tfrac{1}{2}\nabla_a M^I \nabla^a M^J\right)$$
$$0 + \tfrac{1}{24}c_{IJK}e^{-1}\epsilon^{abcde}A^I_a F^J_{bc}F^K_{de} \ .$$

Weyl squared corrected $\mathcal{N} = 2$, $D = 5$ supergravity coupled to Abelian vector multiplets

As far as the four derivative Lagrangian is concerned we will take

$$
\begin{aligned}
\mathcal{L}_4 = \frac{c_{2I}}{24} \Big\{ & \frac{1}{16} e^{-1} \epsilon^{abcde} A^I_a C_{bcfg} C_{de}{}^{fg} + \frac{1}{8} M^I C^{abcd} C_{abcd} + \\
& + \frac{1}{12} M^I D^2 + \frac{1}{6} D v^{ab} F^I_{ab} + \frac{1}{3} M^I C_{abcd} v^{ab} v^{cd} + \frac{1}{2} C_{abcd} F^{Iab} v^{cd} + \\
& + \frac{8}{3} M^I v_{ab} \nabla^b \nabla_c v^{ac} - \frac{16}{9} M^I v^{ab} v_{bc} R_a{}^c - \frac{2}{9} M^I v^2 R + \\
& + \frac{4}{3} M^I \nabla_a v_{bc} \nabla^a v^{bc} + \frac{4}{3} M^I \nabla_a v_{bc} \nabla^b v^{ca} + \\
& - \frac{2}{3} M^I e^{-1} \epsilon^{abcde} v_{ab} v_{cd} \nabla^f v_{ef} + \frac{2}{3} e^{-1} \epsilon^{abcde} F^I_{ab} v_{cf} \nabla^f v_{de} + \\
& + \epsilon^{abcde} F^I_{ab} v_{cf} \nabla_d v_e{}^f - \frac{4}{3} F^I_{ab} v^{ac} v_{cd} v^{db} - \frac{1}{3} F^I_{ab} v^{ab} v_{cd} v^{cd} + \\
& + 4 M^I v_{ab} v^{bc} v_{cd} v^{da} - M^I v_{ab} v^{ab} v_{cd} v^{cd} \Big\} ,
\end{aligned}
$$

EOMs

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta D} = \frac{1}{2}(\mathcal{N}-1)\,, \qquad \frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta v_{\mu\nu}} = 2(\mathcal{N}_I F^{I\mu\nu} + (3\mathcal{N}+1)v^{\mu\nu}),$$

$$\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta M^I} &= (\tfrac{1}{2}D - \tfrac{1}{4}R + 3v^2)\mathcal{N}_I + c_{IJK}(\tfrac{1}{4}F^J\cdot F^K + \tfrac{1}{2}\nabla M^J\cdot\nabla M^K) \\
&\quad +\mathcal{N}_{IJ}(2F^J_{ab}v^{ab} + \nabla^2 M^J) \\
\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta A^I_\mu} &= c_{IJK}(\tfrac{1}{8}\epsilon^{\mu abcd}F^J_{ab}F^K_{cd} + F^{J\mu a}\nabla_a M^K) + 4\mathcal{N}_I\nabla_a v^{\mu a} \\
&\quad +\mathcal{N}_{IJ}(4v^{\mu a}\nabla_a M^J + \nabla_a F^{J\mu a})
\end{aligned}$$

EOMs

$$
\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta g^{\mu\nu}} \;=\;& -\tfrac{1}{4}(\mathcal{N}+3)E_{\mu\nu} - \tfrac{1}{4}D(\mathcal{N}-1)g_{\mu\nu} \\
& +2(1+3\mathcal{N})(v_{a\mu}v^{a}{}_{\nu} - \tfrac{1}{4}v^2 g_{\mu\nu}) \\
& +\mathcal{N}_{IJ}(\tfrac{1}{2}F^{I}_{a\mu}F^{Ja}{}_{\nu} + 4F^{I}_{a(\mu}v^{a}{}_{\nu)} - \tfrac{1}{2}\nabla_{\mu}M^{I}\nabla_{\nu}M^{J}) \\
& -\mathcal{N}_{IJ}(\tfrac{1}{8}F^{I}\cdot F^{J} + F^{I}\cdot v - \tfrac{1}{4}\nabla M^{I}\cdot\nabla M^{J})g_{\mu\nu} \\
& +\tfrac{1}{4}(\nabla_{\mu}\nabla_{\nu}\mathcal{N} - \nabla^2\mathcal{N}g_{\mu\nu})\,.
\end{aligned}
$$

EOMs

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta D} = \frac{c_{2I}}{144}\left\{DM^I + v\cdot F^I\right\}$$

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta M^I} = \frac{c_{2I}}{24}\Big\{\frac{1}{8}C^{abcd}C_{abcd} + \frac{1}{12}D^2 + \frac{1}{3}C_{abcd}v^{ab}v^{cd}$$

$$+\frac{8}{3}v_{ab}\nabla^b\nabla_c v^{ac} - \frac{16}{9}v^{ab}v_{bc}R_a{}^c - \frac{2}{9}v^2R$$

$$+\frac{4}{3}(\nabla_a v_{bc})(\nabla^a v^{bc}) + \frac{4}{3}(\nabla_a v_{bc})(\nabla^b v^{ca})$$

$$-\frac{2}{3}e^{-1}\epsilon^{abcde}v_{ab}v_{cd}\nabla^f v_{ef} + 4v_{ab}v^{bc}v_{cd}v^{da} - (v^2)^2\Big\}$$

EOMs

$$
\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta v_{\mu\nu}} = \tfrac{c_{2I}}{24}\Big\{ & \tfrac{1}{6} D F^{I\mu\nu} + \tfrac{2}{3} M^I C^{\mu\nu}{}_{ab} v^{ab} + \tfrac{1}{2} C^{\mu\nu}{}_{ab} F^{Iab} \\
& + \tfrac{8}{3} M^I \nabla^{[\mu|}\nabla_a v^{|\nu]a} - \tfrac{8}{3}\nabla^{[\mu|}\nabla_a M^I v^{|\nu]a} \\
& + \tfrac{32}{9} M^I v^{[\mu}{}_a R^{\nu]a} - \tfrac{4}{9} M^I R v^{\mu\nu} \\
& - \tfrac{8}{3}\nabla_a M^I \nabla^a v^{\mu\nu} - \tfrac{8}{3}\nabla_a M^I \nabla^{[\mu} v^{\nu]a} \\
& - \tfrac{4}{3} M^I \epsilon^{\mu\nu abc} v_{ab} \nabla^d v_{cd} + \tfrac{2}{3}\epsilon^{abcd[\mu}\nabla^{\nu]} M^I v_{ab} v_{cd} \\
& + \tfrac{2}{3}\epsilon^{abcd[\mu} F^I_{ab} \nabla^{\nu]} v_{cd} - \tfrac{2}{3}\epsilon^{abc\mu\nu}\nabla^d F^I_{ab} v_{cd} \\
& + \epsilon^{abcd[\mu} F^I_{ab} \nabla_c v_d{}^{\nu]} + \epsilon^{abcd[\mu}\nabla_c F^I_{ab} v_d{}^{\nu]} \\
& + \tfrac{8}{3} F^{I[\mu}{}_a v^{\nu]}{}_b v^{ab} - \tfrac{4}{3} F^I_{ab} v^{a\mu} v^{\nu b} - \tfrac{1}{3} v^2 F^{I\mu\nu} \\
& - \tfrac{2}{3}\left(F^I \cdot V\right) v^{\mu\nu} - 16 M^I v_{ab} v^{a\mu} v^{\nu b} - 4 M^I v^2 v^{\mu\nu} \Big\}
\end{aligned}
$$

EOMs

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta A^I_\mu} = \tfrac{c_{2I}}{24}\Big\{\tfrac{1}{16}\epsilon^{\mu abcd}C_{abef}C_{cd}{}^{ef} - \tfrac{1}{3}\nabla_a D v^{a\mu}$$
$$-\nabla_a C^{a\mu}{}_{bc}v^{bc} + \tfrac{4}{3}\epsilon^{\mu abcd}\nabla_a v_{be}\nabla^e v_{cd}$$
$$+2\epsilon^{\mu abcd}\nabla_a v_{be}\nabla_c v_d{}^e + \tfrac{8}{3}\nabla_a v^{ab}v_{bc}v^{c\mu} + \tfrac{2}{3}\nabla_a v^{a\mu}v^2\Big\}$$

EOMs

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta g^{\mu\nu}} = \frac{c_{2I}}{24}\Bigg\{ \quad -\tfrac{1}{8}\left[\epsilon^{abcd}{}_{(\mu|}\nabla_e F^I_{ab} R_{cd}{}^e{}_{|\nu)}\right]$$

$$+\tfrac{1}{4}\Big[M^I\Big(-C_{abc(\mu|}R^{abc}{}_{|\nu)} + \tfrac{4}{3}R_{ab}C_\mu{}^a{}_\nu{}^b + 2C_\mu{}^{bcd}C_{\nu bcd}$$

$$-\tfrac{1}{4}g_{\mu\nu}C^{abcd}C_{abcd}\Big) + 2\nabla_a\nabla_b M^I C^a{}_\mu{}^b{}_\nu\Big]$$

$$-\tfrac{1}{24}\left[g_{\mu\nu}M^I D^2\right] + \tfrac{1}{3}\left[Dv_{(\mu}{}^a F^I_{\nu)a} - \tfrac{1}{4}g_{\mu\nu}Dv^{ab}F^I_{ab}\right]$$

$$+\tfrac{1}{3}\Big[M^I\Big(\left(R_{abc(\mu} - 4C_{abc(\mu}\right)v^{ab}v_{\nu)}{}^c + \tfrac{4}{3}R_{ab}v_\mu{}^a v_\nu{}^b - \tfrac{1}{3}Rv_\mu{}^a v_{\nu a}$$

$$+\tfrac{1}{6}R_{\mu\nu}v^{ab}v_{ab} - \tfrac{1}{2}g_{\mu\nu}C_{abcd}v^{ab}v^{cd}\Big)$$

$$+ 2\nabla_a\nabla_b v_\mu{}^a v_\nu{}^b M^I + \tfrac{4}{3}\nabla_a\nabla_{(\mu}v_{\nu)b}v^{ab}M^I$$

$$-\tfrac{2}{3}\nabla^2 v_\mu{}^a v_{\nu a}M^I + \tfrac{2}{3}g_{\mu\nu}\nabla_a\nabla_b v^{ac}v_c{}^b M^I$$

$$+ \tfrac{1}{6}\left(g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu\right)v^{ab}v_{ab}M^I\Big]$$

EOMs

$$
\begin{aligned}
&+ \Big[\tfrac{1}{2} R_{abc(\mu} v_{\nu)}{}^{c} F^{lab} + \nabla_a \nabla_b v_{(\mu}{}^{a} F^{l}_{\nu)}{}^{b} + \tfrac{1}{3} \nabla_a \nabla_{(\mu|} v_{|\nu)b} F^{lab} \\
&+ \tfrac{1}{3} \nabla_a \nabla_{(\mu} F^{lb}{}_{\nu)} v_b{}^{a} + \tfrac{1}{3} \nabla^2 F^{la}{}_{(\mu} v_{\nu)a} \\
&\quad - \tfrac{1}{3} g_{\mu\nu} \nabla_a \nabla_b v^{a}{}_{c} F^{lbc} + \tfrac{2}{3} R_{ab} F^{la}{}_{(\mu} v^{b}{}_{\nu)} \\
&+ \tfrac{1}{12} \left(R_{\mu\nu} - \nabla_\mu \nabla_\nu + g_{\mu\nu} \nabla^2\right) v_{ab} F^{lab} \\
&\quad + \tfrac{1}{6} R F^{la}{}_{(\mu} v_{\nu)a} - \left(F^{la}{}_{(\mu} v^{bc} + v^{a}_{(\mu} F^{lbc}\right) C_{|\nu)abc} \\
&- \tfrac{1}{4} g_{\mu\nu} F^{lab} v^{cd} C_{abcd}\Big] \\
&+ \tfrac{8}{3} \Big[M^{l} \Big(v_{a(\mu} \nabla_{\nu)} \nabla_b v^{ab} + v_{ab} \nabla^{b} \nabla_{(\mu} v^{a}{}_{\nu)} + v_{(\mu|}{}^{a} \nabla_a \nabla_b v_{|\nu)}{}^{b} \\
&- \tfrac{1}{2} g_{\mu\nu} v_{ab} \nabla^{b} \nabla_c v^{ac}\Big) + \nabla_a v_{(\mu|}{}^{a} \nabla_b M^{l} v_{|\nu)}{}^{b} \\
&\quad - \nabla_{(\mu} v_{\nu)a} \nabla_b M^{l} v^{ab} + \tfrac{1}{2} g_{\mu\nu} \nabla_a v^{a}{}_{b} \nabla_c M^{l} v^{bc} - \nabla_a M^{l} v^{ab} \nabla_{(\mu} v_{\nu)b}\Big] \\
&- \tfrac{16}{9} \Big[M^{l} \Big(v^{a}{}_{\mu} v_{\nu}{}^{b} R_{ab} - 2 v^{ab} v_{a(\mu} R_{\nu)b} - \tfrac{1}{2} g_{\mu\nu} v^{ab} v_{b}{}^{c} R_{ac}\Big)
\end{aligned}
$$

EOMs

$$+\frac{1}{2}\nabla^2 M^I v_{(\mu|}{}^{a} v_{a|\nu)}$$

$$+\frac{1}{2} g_{\mu\nu}\nabla_a\nabla_b M^I v^{ac} v_c{}^{b} - \nabla_a\nabla_{(\mu|} M^I v^{ab} v_{b|\nu)}\Big]$$

$$-\frac{2}{9}\Big[M^I\Big(2 v_\mu{}^{a} v_{\nu a} R + v_{ab} v^{ab} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R v_{ab} v^{ab}\Big)$$

$$-\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2\right) M^I v_{ab} v^{ab}\Big]$$

$$+\frac{4}{3}\Big[M^I\Big((\nabla_\mu v_{ab})(\nabla_\nu v^{ab}) + 2(\nabla_a v_{b\mu})(\nabla^a v^b{}_\nu) - \frac{1}{2} g_{\mu\nu}(\nabla_a v_{bc})(\nabla^a v^{bc})\Big)$$

$$+2\nabla_a M^I(\nabla^a v_{(\mu|}{}^{b}) v_{b|\nu)} + 2\nabla_a M^I(\nabla_{(\mu|} v^{ab}) v_{b|\nu)}$$

$$-2\nabla_a M^I(\nabla_{(\mu|} v_{b|\nu)})) v^{ab}\Big]$$

$$+\frac{4}{3}\Big[M^I\Big(2(\nabla_{(\mu|} v^{ab})(\nabla_a v_{b|\nu)}) + (\nabla_a v_{b(\mu|})(\nabla^b v_{|\nu)}{}^{a})$$

$$-\frac{1}{2} g_{\mu\nu}(\nabla_a v_{bc})(\nabla^b v^{ca})\Big)$$

$$+\nabla_a\Big(M^I v_{b(\mu}\nabla_{\nu)} v^{ba} + M^I v_{b(\mu}\nabla^a v^b{}_{\nu)} - M^I v^{ba}\nabla_{(\mu|} v_{b|\nu)}\Big)\Big]$$

EOMs

$$
\begin{aligned}
&- \tfrac{2}{3}\Big[M^I \epsilon^{abcde} v_{ab} v_{cd} \nabla_{(\mu|} v_{e|\nu)} - \epsilon^{abcde} \nabla_{(\mu|} M^I v_{ab} v_{cd} v_{e|\nu)} \\
&\qquad - \epsilon^{abcd}{}_{(\mu|} \nabla_e M^I v_{ab} v_{cd} v_{|\nu)}{}^e + \tfrac{1}{2} g_{\mu\nu} \epsilon^{abcde} \nabla^f M^I v_{ab} v_{cd} v_{ef}\Big] \\
&+ \tfrac{2}{3}\Big[\epsilon^{abcde} F^I_{ab} v_{c(\mu} \nabla_{\nu)} v_{de} - 2\epsilon^{abcd}{}_{(\mu|} \nabla_e F^I_{ab} v_c{}^e v_{d|\nu)}\Big] \\
&+ \Big[\epsilon^{abcde} F^I_{ab} v_{c(\mu|} \nabla_d v_{e|\nu)} + \epsilon^{abcd}{}_{(\mu|} \nabla_e F^I_{ab} v_c{}^e v_{d|\nu)}\Big] \\
&- \tfrac{4}{3}\Big[2F^I_{a(\mu} v_{\nu)}{}^b v_{bc} v^{ac} - 2F^I_{ab} v^a{}_{(\mu} v_{\nu)c} v^{bc} - \tfrac{1}{2} g_{\mu\nu} F^I_{ab} v^{ac} v_{cd} v^{db}\Big] \\
&- \tfrac{1}{3}\Big[2F^I_{a(\mu} v^a{}_{\nu)} v_{bc} v^{bc} + 2F^{Iab} v_{ab} v_{c\mu} v^c{}_\nu - \tfrac{1}{2} g_{\mu\nu} F^{Iab} v_{ab} v^{cd} v_{cd}\Big] \\
&+ \Big[16 M^I v_{ab} v^b{}_{(\mu} v_{\nu)c} v^{ca} - 2 g_{\mu\nu} M^I v_{ab} v^{bc} v_{cd} v^{da}\Big] \\
&+ \Big[4 M^I v_{ab} v^{ab} v_{c\mu} v_\nu{}^c + \tfrac{1}{2} g_{\mu\nu} M^I v_{ab} v^{ab} v_{cd} v^{cd}\Big]\Big\}
\end{aligned}
$$

Conditions for timelike supersymmetry

- For a timelike orbit we can take $\epsilon = (\epsilon^1, \epsilon^2) = (e^{\phi}1, -ie^{\phi}e^{12})$

The Killing spinor equations imply

- U(1) fibration of Hyper-Kahler base space
- $ds^2 = e^{4\phi}(dt+\Omega)^2 - e^{-2\phi}\hat{g}_{mn}dx^m dx^n$
- $F^I = e^{-2\phi}e^0 \wedge d(M^I e^{2\phi}) - M^I G^{(-)} + F^{I(+)} = -d(M^I e^0) + \Theta^I$
- Θ harmonic.
- $v_{\mu\nu}$ completely determined.
- $D = \frac{3}{2}e^{4\phi}\hat{G}^{(-)}\cdot\hat{G}^{(-)} + \frac{1}{2}e^{4\phi}\hat{G}^{(+)}\cdot\hat{G}^{(+)} + 3e^{2\phi}\hat{\nabla}^2\phi - 18e^{2\phi}(\hat{\nabla}\phi)^2$

EOMs and KSIs

Normally to solve the eoms we plug this data into the equations then try and simplify. However for our representative we obtain for the Killing spinor identities

$$
\begin{aligned}
\mathcal{E}(A)^0_I - \mathcal{E}(M)_I = 0 \ , \qquad \mathcal{E}(A)^i_I &= 0 \ , \\
\left(\tfrac{1}{4}\mathcal{E}(v) + \mathcal{E}(D)v\right)^{\alpha}{}_{\alpha} + \nabla^0\mathcal{E}(D) &= 0 \ , \\
\left(\tfrac{1}{4}\mathcal{E}(v) + \mathcal{E}(D)v\right)^{0i} - \nabla^i\mathcal{E}(D) &= 0 \ , \\
\left(\tfrac{1}{4}\mathcal{E}(v) + \mathcal{E}(D)v\right)^{12} = 0 \ , \qquad \mathcal{E}(e)^{\mu}_a &= 0 \ .
\end{aligned}
$$

Note that as the KSI are a consequence of the off-shell supersymmetry, these are valid for all higher order corrections that can be added to the theory with the same field content, i.e. for any consistent truncation in which the SU(2) triplet fields are set to zero. In particular for any such corrected action, including the one under consideration, it is sufficient to impose the equations of motion

$$
\mathcal{E}(D) = 0 \ , \qquad \mathcal{E}(v)^{(+)ij} = 0 \ , \qquad \mathcal{E}(M)_I = 0 \ .
$$

Simplified eoms

The equation of motion for D is

$$0 = \frac{1}{2}(\mathcal{N} - 1) + \frac{c_{2I}}{48} e^{2\phi} \left[\frac{1}{4} e^{2\phi} M^I \left(\frac{1}{3} \hat{G}^{(+)} \cdot \hat{G}^{(+)} + \hat{G}^{(-)} \cdot \hat{G}^{(-)} \right) \right.$$
$$\left. + \frac{1}{12} e^{2\phi} \hat{G}^{(+)} \cdot \hat{\Theta}^{(+)I} + M^I \hat{\nabla}^2 \phi + \hat{\nabla}\phi \cdot \hat{\nabla} M^I - 4 M^I \hat{\nabla}\phi \cdot \hat{\nabla}\phi \right] ,$$

The M^I equation is more involved, but we find

$$0 = e^{4\phi} \left[\frac{1}{4} c_{IJK} \hat{\Theta}^{(+)J} \cdot \hat{\Theta}^{(+)K} - \hat{\nabla}^2 \left(e^{-2\phi} \mathcal{N}_I \right) \right] +$$
$$+ \frac{c_{2I}}{24} e^{4\phi} \left\{ \hat{\nabla}^2 \left(3 \hat{\nabla}\phi \cdot \hat{\nabla}\phi - \frac{1}{12} e^{2\phi} \hat{G}^2_{(+)} - \frac{1}{4} e^{2\phi} \hat{G}^2_{(-)} \right) + \frac{1}{8} \hat{R}_{ijkl} \hat{R}^{ijkl} \right\} ,$$

This computation has been checked in Mathematica using the package xAct , and the two equations above are in agreement with [Castro et al].

Simplified eoms

Finally, after a very long calculation and making extensive use of duality identities we find the equation of motion for v yeilds

$$
\begin{aligned}
0 = \quad & -4e^{2\phi}\hat{G}^{(+)}_{ij} + 2e^{2\phi}\mathcal{N}_I\hat{\Theta}^{I(+)}_{ij} \\
& + \frac{c_{2I}}{24}\left\{\frac{1}{2}e^{6\phi}\left(\frac{1}{3}\hat{G}^2_{(+)} + \hat{G}^2_{(-)}\right)\hat{\Theta}^{(+)I}_{ij}\right. \\
- \quad & \frac{1}{3}e^{4\phi}\left(M^I\hat{G}^{(+)}_{kl} + 2\hat{\Theta}^{I(+)}_{kl}\right)\hat{R}_{ij}{}^{kl} \\
& + e^{4\phi}\hat{\nabla}^2\left[M^I(G^{(-)}_{ij} - \frac{1}{3}G^{(+)}_{ij})\right] - \frac{1}{6}e^{-2\phi}\hat{\nabla}^2[e^{6\phi}\hat{\Theta}^{I(+)}_{ij}] \\
- \quad & \left. 4e^{4\phi}\hat{\nabla}_{[i}\hat{\nabla}_k[M^I G^{(-)k}{}_{j]}]\right\} ,
\end{aligned}
$$

- Supersymmetric solutions of supergravities have their uses!
- We have the methods to finish the classifications in the two derivate cases and to all orders in some cases.
- Higher derivative corrections in supergravities allow us to do calculations which include stringy effects.
- String (very very early universe) cosmology, black hole physics and supersymmetry phenomonology are the most promising directions in order to try to test and develop string theory.
- If you are interested in studying these topics for a thesis write to me at psloane@mctp.mx

Thanks for your attention.
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