Un sabor de la teoría de cuerdas

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En colaboración con B. Carballo-Pérez & E. Peinado: arXiv:1607.06812

Almost accepted

Referee report

The authors of JHEP_061P_0816 have presented a novel synthesis of "top-down" and "bottom-up" approaches to motivate a string-inspired flavor model with the finite symmetry $\Delta(54)$. Although the model presented is preliminary and incomplete (Eq. 17), it leads to an interesting correlation between the atmospheric and reactor angles in the neutrino sector, and definite predictions for the neutrino parameters at the edges of their best fit values which could soon be ruled out by more precise measurements. As the article contains falsifiable predictions for the neutrino parameters and a well motivated UV completion which restricts some of the arbitrariness of the model-building, I believe it to be suitable for publication in JHEP upon minor revisions and the correspondence of the authors on the following points:

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  - three flavors of SM particles
  - observed mass patterns
  - CKM, PMNS phases
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  - suppression of proton decay
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  Observation: $\mathbb{Z}_3, S_3, \Delta(27), \Delta(54) \subset \mathrm{SU}(3)$

Flavor symmetries: bottom–up

Flavor Symmetries = Discrete symmetries between generations in $\mathcal{L}$

$$G = D_4, S_4, A_4, D_5, P_6, \ldots, \mathbb{Z}_3, S_3, \Delta(27), \ldots$$

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Unfortunately

$\Delta(54)$ is almost unexplored! ☹

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Our purpose HERE:

Explore potential of $\Delta(54)$ and its origin ☺

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  additional discrete symmetries due to **compact dimensions**

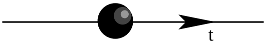
geometry $\rightarrow$ symmetry

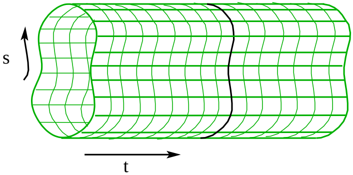
Discrete non-Abelian flavor symmetries can arise from the geometric structure of string compactifications! ☺

Strings

Strings

1970's:





1980's: Superstring theories (SUSY + strings)

- type I
- type IIA
- type IIB
- Heterotic $\mathrm{E}_8 \times \mathrm{E}_8$
- Heterotic $\mathrm{SO}(32)$

$\rightarrow$ gauge bosons + 10D $\mathcal{N} = 1$ SUSY

$\longrightarrow$ includes gravity; no anomalies (nor tachyons)

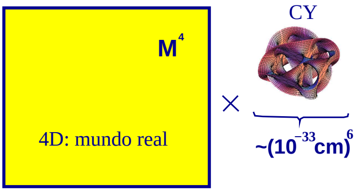
Compactifying string theories

- compactifications:

$$\mathcal{M}^{9,1} = \mathcal{M}^{3,1} \otimes X_6 \qquad \text{vol}(X_6) \sim \ell_{Pl}^6, \quad \mathcal{N} = 1$$

1. X_6: Calabi-Yau threefolds

Candelas,Horowitz,Strominger,Witten (1985)



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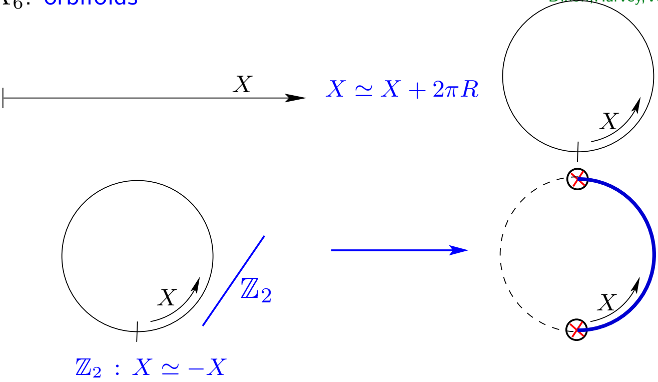

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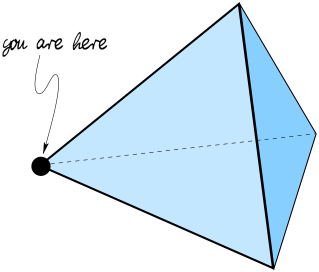

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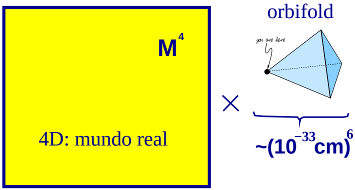

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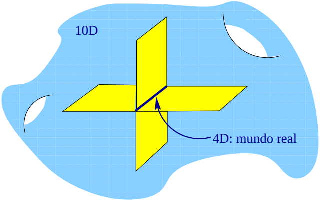
- D-brane worlds (in type I & II):



Framework: toroidal orbifolds

Orbifold:

$$\mathcal{O} = \frac{\text{compact manifold } \mathcal{M}}{\text{discrete group of isometries } I}$$

Toroidal Orbifold:

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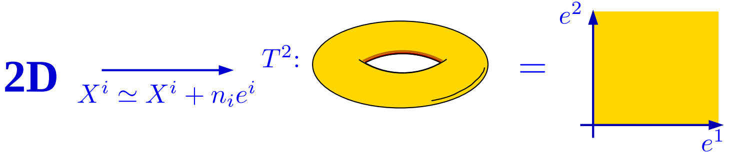
E.g. 2D torus T^2



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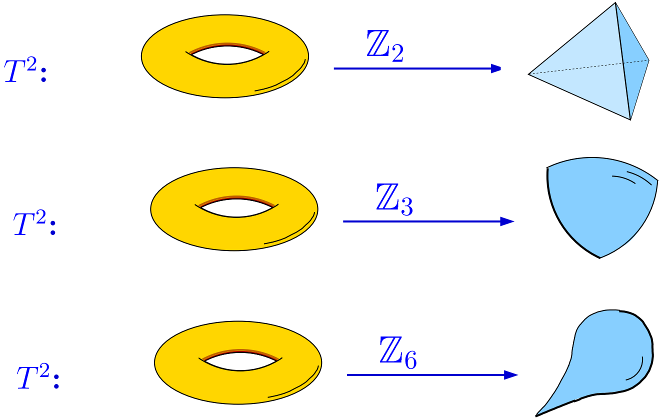
$$I = \begin{cases} \text{Abelian, e.g. } \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_3 \times \mathbb{Z}_3 \\ \text{non-Abelian} \end{cases}$$

$$g \in I \quad \Rightarrow \quad g = (\Theta, v) \quad \text{such that} \quad gX = \Theta X + v$$

$$\text{where} \qquad \Theta = \vartheta^p \omega^q \quad \text{for } \mathbb{Z}_N \times \mathbb{Z}_M$$

2D toroidal orbifolds and fixed points

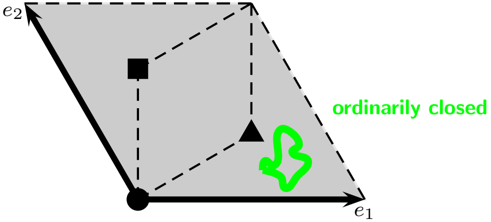
2D $\mathbb{Z}_N$ orbifolds



Possible closed strings on the orbifold

Three types of *closed strings*:

ordinarily closed, closed on the torus, closed under the orbifold



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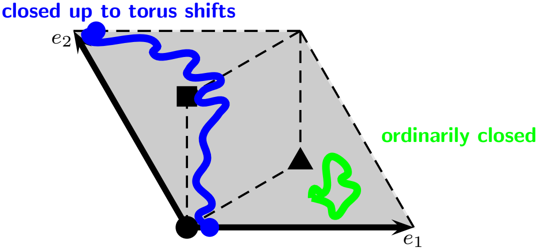

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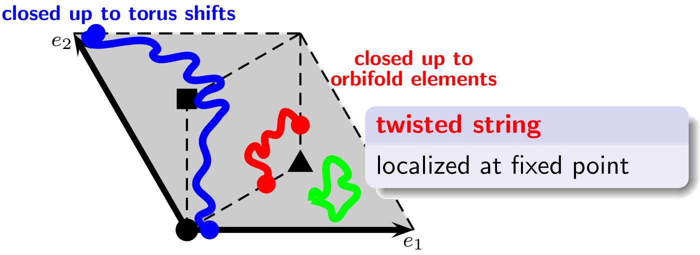

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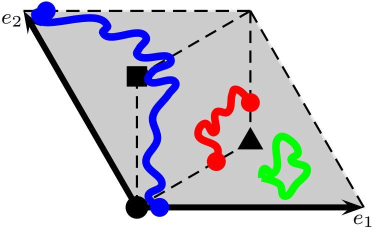

Twisted strings located at fixed points $\longrightarrow$ LEEF states localized

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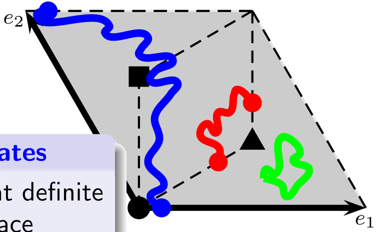

localized twisted states

LEEF states appear at definite points in compact space

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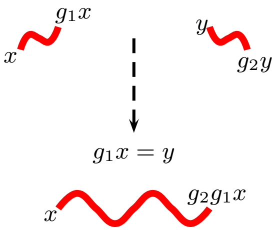

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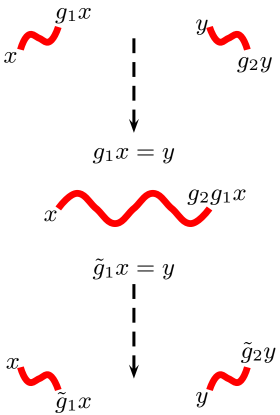

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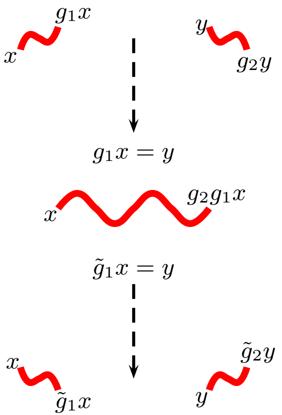

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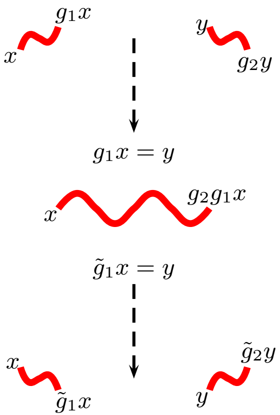

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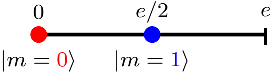
Stringy space group selection rule

$\iff$ LEEF symmetry (Z) of the couplings, acting on the charges p, n_α.

- See how this works!

Flavor symmetry for $S^1/\mathbb{Z}_2$, $\vartheta = -1$

space group rule



relabeling

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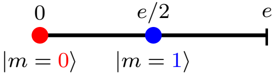

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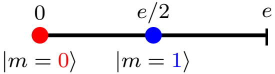

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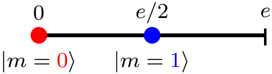

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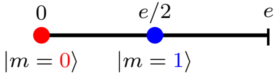

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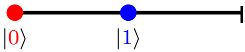
degenerate fixed points (no Wilson lines)



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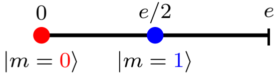

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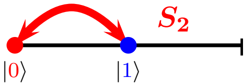
space group rule $\mathbb{Z}_2$'s:

$$\boldsymbol{\Phi \longmapsto \{\sigma_3, -\mathbb{1}\}\Phi}$$

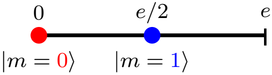
relabeling

degenerate fixed points (no Wilson lines)



Flavor symmetry for $S^1/\mathbb{Z}_2$, $\vartheta = -1$

space group rule



$$\prod_{j=1}^{n}(\vartheta^{p^{(j)}}, m^{(j)}e) = (\vartheta^{\sum_j p^{(j)}}, (\textstyle\sum_j m^{(j)})e)$$

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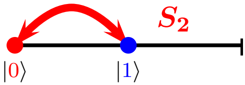

$\Rightarrow$ relabeling symmetry S_2:

(permutation symmetry)

$$m \mapsto (m+1) \mod 2$$

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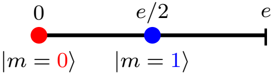

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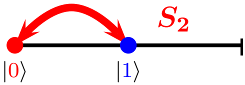

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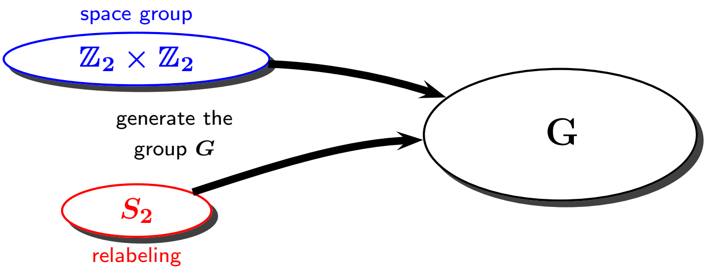

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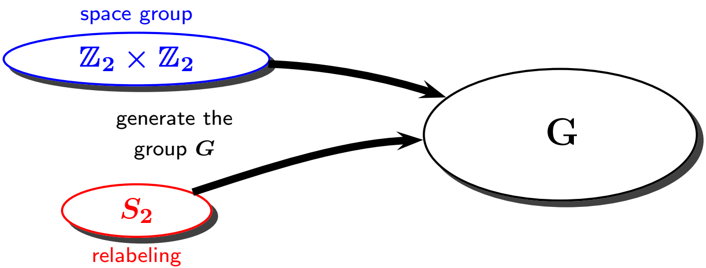

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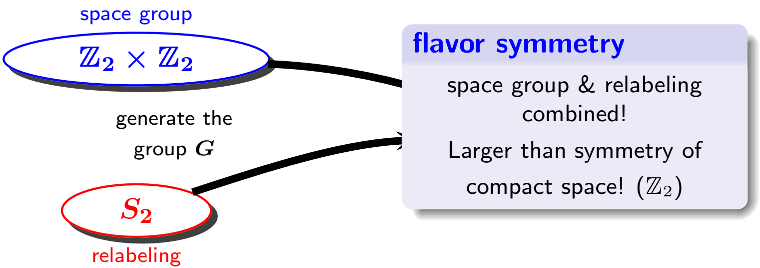

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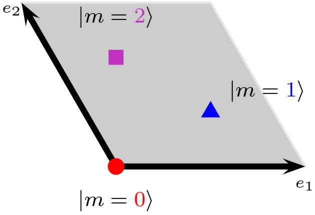

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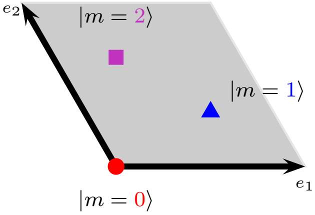

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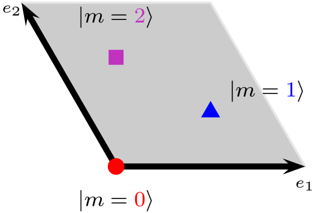

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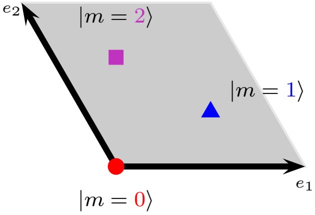

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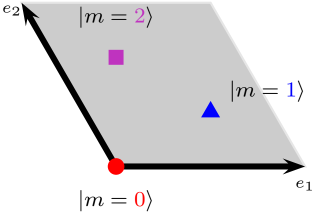

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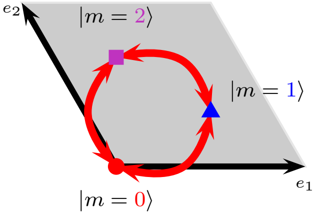

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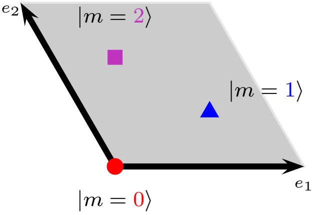

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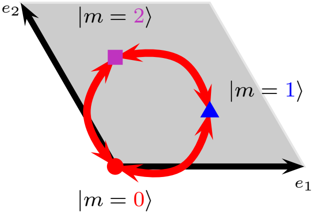

$\Downarrow$

relabeling symmetry:

S_3

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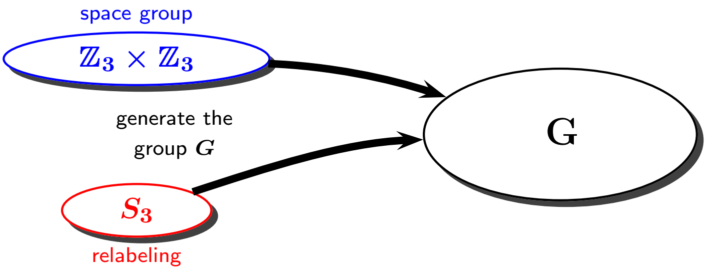

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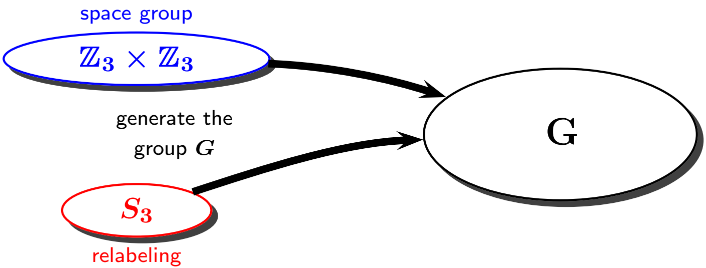

$\mathbb{Z}_3 \times \mathbb{Z}_3$ (Abelian) $\cup$ S_3 (non-Abelian) $=$ G **non-Abelian!**

- There is a *normal subgroup* $N = \mathbb{Z}_3 \times \mathbb{Z}_3 \subset G$

$$G = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3) = \Delta(54)$$

$\Delta(54)$ flavor symmetry in 6D orbifolds

- Construction kit:
  1. specify 6D geometry Λ, I
  2. $\left.\begin{array}{lcc}\text{space group rule} & \rightarrow & Z \\ \text{relabeling} & \rightarrow & S\end{array}\right\} \longrightarrow S \cup Z$
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- BUT no promising models in $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_2$ with $\Delta(54)$

Lebedev,Ratz,SR-S,Vaudrevange (2008)

Investigate $\mathbb{Z}_3 \times \mathbb{Z}_3$ orbifolds!

Flavor symmetries in $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- If all fixed points are degenerate, flavor symmetry is large:

$$G = S_3 \times S_3 \times S_3 \ltimes \left[\mathbb{Z}_3^{\vartheta} \times \mathbb{Z}_3^{\omega} \times (\mathbb{Z}_3^{m})^3\right]$$

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Procedure to study stringy models with $\Delta(54)$

1. Determine all (inequivalent) $\mathrm{E}_8\times\mathrm{E}_8$ gauge embeddings in heterotic string

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2. Select semi-realistic models
   - 4D gauge group $= \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times G_{hidden}$
   - 3 generations of quarks and leptons + a pair H_u, H_d
   - $\sin^2\theta_w(M_{GUT}) = 3/8$
   - only SM-vectorlike extra matter

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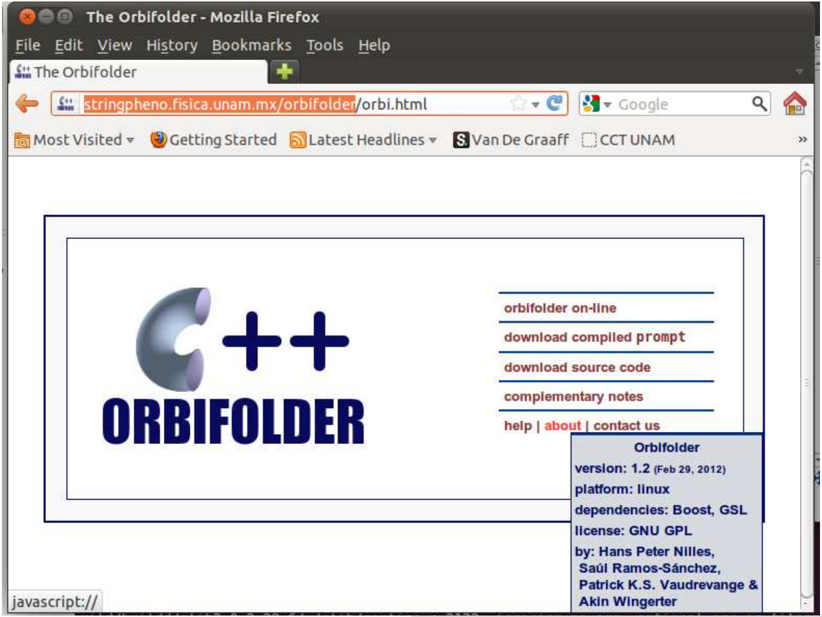
7. Study the low-energy consequences for quarks and leptons

$\to$ predictions?

Orbifolder needed as a tool (Nilles, SR-S, Vaudrevange, Wingerter, 2011)



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We found over 7×10^6 admissible gauge embeddings:

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* Numbers compatible with Nilles,Vaudrevange (2014), but we find many more models ☺

$\Delta(54)$ stringy example

Example

$\Delta(54)$ string spectrum

#	irrep	$\Delta(54)$	label	#	anti-irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i				
3	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\bar{u}_i$				
3	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\bar{d}_i$				
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i				
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\bar{e}_i$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\bar{\nu}_i$				
1	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{1}_{0}$	H_d	1	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\mathbf{1}_{0}$	H_u
			Flavons				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	ϕ_i^u				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	$\phi_i^{d,e}$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	ϕ_i^{ν}				
2	$(\mathbf{1},\mathbf{1})_{0}$	$2\cdot\mathbf{1}_{0}$	$s^{(d,e)}$, s^u				
128	$(\mathbf{1},\mathbf{1})_{0}$	$77\cdot\mathbf{1}_0+16\cdot\mathbf{3}_{12}+\mathbf{3}_{11}$	N_i				
			Exotic states				
16	$(\mathbf{1},\mathbf{2})_{\frac{1}{6}}$	$10\cdot\mathbf{1}_0+2\cdot\mathbf{3}_{12}$	v_i	16	$(\mathbf{1},\mathbf{2})_{-\frac{1}{6}}$	$4\cdot\mathbf{1}_0+4\cdot\mathbf{3}_{12}$	$\bar{v}_i$
3	$(\mathbf{3},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	y_i	3	$(\overline{\mathbf{3}},\mathbf{1})_{0}$	$3\times\mathbf{1}_0$	$\bar{y}_i$
1	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	$\mathbf{1}_{0}$	z_i	1	$(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{1}_0$	$\bar{z}_i$
7	$(\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$4\cdot\mathbf{1}_0+\mathbf{3}_{12}$	x_i	7	$(\mathbf{1},\mathbf{1})_{\frac{2}{3}}$	$4\cdot\mathbf{1}_0+\mathbf{3}_{11}$	$\bar{x}_i$
51	$(\mathbf{1},\mathbf{1})_{-\frac{1}{3}}$	$30\cdot\mathbf{1}_0+7\cdot\mathbf{3}_{12}$	w_i	51	$(\mathbf{1},\mathbf{1})_{\frac{1}{3}}$	$24\cdot\mathbf{1}_0+9\cdot\mathbf{3}_{12}$	$\bar{w}_i$

$$W_Y = y^u_{ijk}Q_iH_u\bar{u}_j\phi^u_k s_u + y^d_{ijk}Q_iH_d\bar{d}_j\phi^{(d,e)}_k s^{(d,e)} + y^e_{ijk}L_iH_d\bar{e}_j\phi^{(d,e)}_k s^{(d,e)}$$
$$+\ y^{\nu}_{ijkl}L_iH_u\bar{\nu}_j + \lambda_{ijk}\bar{\nu}_i\bar{\nu}_j\bar{\phi}^{\nu}_k\,, \qquad i,j,k=1,2,3,$$

$\Delta(54)$ Phenomenology

$\Delta(54)$ stringy phenomenology: down sector

From the superpotential W_Y, quark and charged-lepton Lagrangian is

$$\begin{aligned}\mathcal{L}_Y^f &= y_1^f\left[F_1 H \bar{f}_1 \phi_1 + F_2 H \bar{f}_2 \phi_2 + F_3 H \bar{f}_3 \phi_3\right] \\ &+ y_2^f\left[(F_1 H \bar{f}_2 + F_2 H \bar{f}_1)\phi_3 + (F_3 H \bar{f}_1 + F_1 H \bar{f}_3)\phi_2 + (F_2 H \bar{f}_3 + F_3 H \bar{f}_2)\phi_1\right] + h.c.\,,\end{aligned}$$

F_i : LH fermion, f_i : RH fermion, H : Higgs, ϕ_i : flavon scalar

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$\Rightarrow$ down-quark and charged-lepton masses

$$M_f^D = \begin{pmatrix} y_1^f\phi_1^f & y_2^f\phi_3^f & y_2^f\phi_2^f \\ y_2^f\phi_3^f & y_1^f\phi_2^f & y_2^f\phi_1^f \\ y_2^f\phi_2^f & y_2^f\phi_1^f & y_1^f\phi_3^f \end{pmatrix} \qquad f = u, d, e$$

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assume a vacuum/flavon alignment: $\langle \phi^f \rangle = v_\phi^f (0, r^f, 1)$

$$M_f^D = \begin{pmatrix} 0 & a^f & a^f r^f \\ a^f & b^f r^f & 0 \\ a^f r^f & 0 & b^f \end{pmatrix} \qquad a^f \equiv y_2^f v_\phi^f \quad \& \quad b^f \equiv y_1^f v_\phi^f$$

$\Delta(54)$ stringy phenomenology: down sector

impose now the mass invariants (m_i^f: measured fermion masses)

$$
\begin{aligned}
\operatorname{tr} M_f^D &= b^f\,(1+r^f) &&\overset{!}{=} -m_1^f + m_2^f + m_3^f\,,\\
\operatorname{tr}(M_f^D)^2 &= [2(a^f)^2+(b^f)^2][1+(r^f)^2] &&\overset{!}{=} (m_1^f)^2+(m_2^f)^2+(m_3^f)^2\,,\\
\det M_f^D &= -(a^f)^2 b^f\,[1+(r^f)^3] &&\overset{!}{=} -m_1^f m_2^f m_3^f\,,
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\end{aligned}$$

taking the hierarchical sol: $r^f \approx (m_2^f - m_1^f)/m_3^f \ll 1\,,\ (a^f)^2 \approx m_1^f m_2^f\,,\ b^f \approx m_3^f$

$$M_f^D \approx \begin{pmatrix} 0 & \sqrt{m_1^f m_2^f} & \frac{m_2^f - m_1^f}{m_3^f}\sqrt{m_1^f m_2^f} \\ \sqrt{m_1^f m_2^f} & m_2^f - m_1^f & 0 \\ \frac{m_2^f - m_1^f}{m_3^f}\sqrt{m_1^f m_2^f} & 0 & m_3^f \end{pmatrix},$$

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For down-quarks

$$\Rightarrow \quad \tan\theta_C \approx \frac{(M_d^D)_{12}}{(M_d^D)_{22}} \approx \sqrt{\frac{m_d}{m_s}}$$

Gatto-Sartori-Tonin ☺

$\Delta(54)$ stringy phenomenology: down sector

Down-quark and charged-lepton sectors are symmetric

$$\Rightarrow \quad \tan\theta_C^e \approx \frac{(M_e^D)_{12}}{(M_e^D)_{22}} \approx \sqrt{\frac{m_e}{m_\mu}} \qquad \text{leptonic Gatto-Sartori-Tonin} \quad ☺$$

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BUT all other quark and charged-lepton mixing angles are smaller than observed ☹

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Additional novel consequence

$$\frac{m_s - m_d}{m_b} \overset{!}{=} \frac{m_\mu - m_e}{m_\tau}.$$

more stringent than $b-\tau$ unification ☹

$\Delta(54)$ stringy phenomenology: neutrino sector

From the superpotential W_Y, renormalizable neutrino Lagrangian is

$$\begin{aligned}\mathcal{L}_Y^\nu &= y_1^\nu \left[L_1 H_u \bar\nu_1 + L_2 H_u \bar\nu_2 + L_3 H_u \bar\nu_3\right] \\ &+ \lambda_1 \left[\bar\nu_1\bar\nu_1\bar\phi_1^\nu + \bar\nu_2\bar\nu_2\bar\phi_2^\nu + \bar\nu_3\bar\nu_3\bar\phi_3^\nu\right] \\ &+ \lambda_2 \left[2\bar\nu_1\bar\nu_2\bar\phi_3^\nu + 2\bar\nu_1\bar\nu_3\bar\phi_2^\nu + 2\bar\nu_2\bar\nu_3\bar\phi_1^\nu\right]\end{aligned}$$

$\Rightarrow$ type I see-saw possible!! ☺

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$\Rightarrow$ RH neutrino masses masses

$$M_{RH} = \begin{pmatrix} \lambda_1\bar\phi_1^\nu & \lambda_2\bar\phi_3^\nu & \lambda_2\bar\phi_2^\nu \\ \lambda_2\bar\phi_3^\nu & \lambda_1\bar\phi_2^\nu & \lambda_2\bar\phi_1^\nu \\ \lambda_2\bar\phi_2^\nu & \lambda_2\bar\phi_1^\nu & \lambda_1\bar\phi_3^\nu \end{pmatrix}$$

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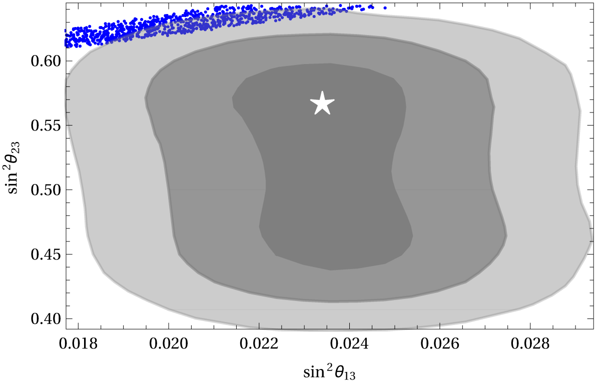
assume a vacuum/flavon alignment: $\langle\bar\phi^\nu\rangle = v_{\nu_3}\,(R_1,\delta,1)$

$$M_\nu = \lambda \begin{pmatrix} \delta - R^2R_1^2 & R(-1+RR_1\delta) & R(-\delta^2+RR_1) \\ R(-1+RR_1\delta) & R_1 - R^2\delta^2 & R(R\delta - R_1^2) \\ R(-\delta^2+RR_1) & R(R\delta-R_1^2) & R_1\delta - R^2 \end{pmatrix}, \quad R = \frac{\lambda_2}{\lambda_1}, \; \lambda = \frac{y_1^2\langle H_u\rangle^2}{function(R_1,R,\delta)}$$

$\Delta(54)$ stringy phenomenology: neutrino sector

Correlation atmospheric–reactor mixing angles compatible with best fit

Forero, Tortola, Valle (2014)



- atmospheric mixing angle: $51.3^o \lesssim \theta_{23} \lesssim 53.1^o$ (second octant) ☺
- reactor mixing angle: $7.8^o \lesssim \theta_{12} \lesssim 8.9^o$ ☺
- $6meV \lesssim m_{\nu_1} \lesssim 6.8meV\,,\quad 65meV \lesssim \sum m_\nu \lesssim 70meV$ ☺

$\Delta(54)$ stringy phenomenology: neutrino sector

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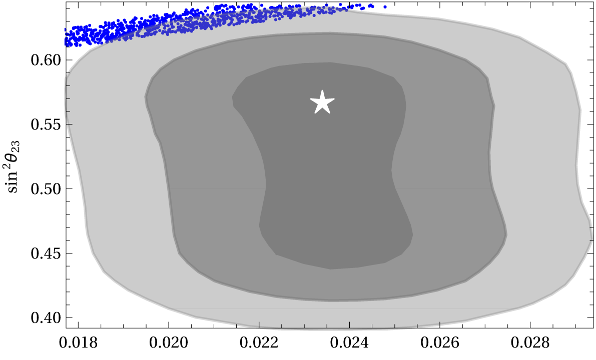


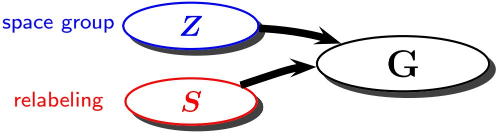
Advantages of model:

Compatible with T2K (2016)
only normal hierarchy admissible
Falsifiable soon

- atmospheric … cond octant) ☺
- reactor mixin…
- $6meV \lesssim m_{\nu_1}$ … eV ☺

Conclusions

- $\Delta(54)$ (and other) flavor symmetries from compactification geometry:



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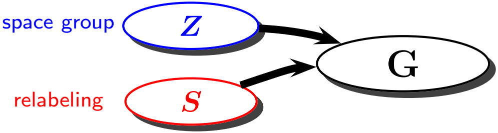

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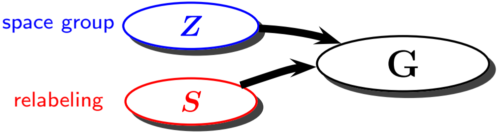

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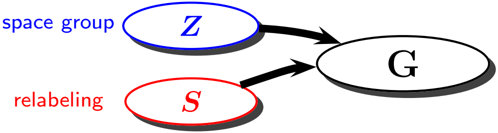

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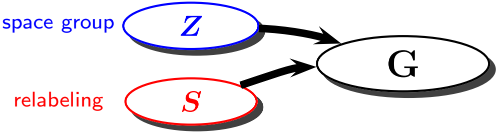

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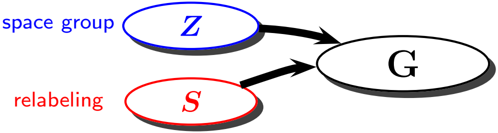
- Leading to
  1. $6meV \lesssim m_{\nu_1} \lesssim 6.8meV, \quad 65meV \lesssim \sum m_\nu \lesssim 70meV$ ☺
  2. correct masses for quarks and leptons
  3. Gatto-Sartori-Tonin in down-sector
  4. funny unification mass relation ☹
  5. normal hierarchy for neutrino masses
  6. "predict" neutrino mixing angles ☺

Conclusions

- $\Delta(54)$ (and other) flavor symmetries from compactification geometry:



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BUT

- bad quark mixing angles
- proton decay
- unjustified vacuum alignments
- SUSY breaking not understood