



BUAP

La búsqueda con álgebra lineal

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Álgebra ... Lineal

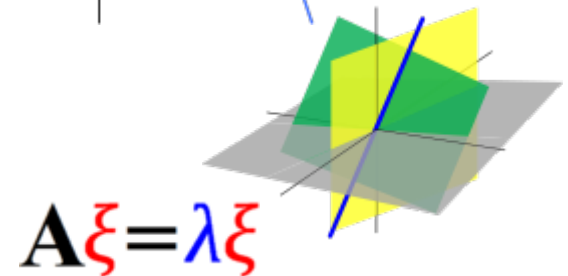
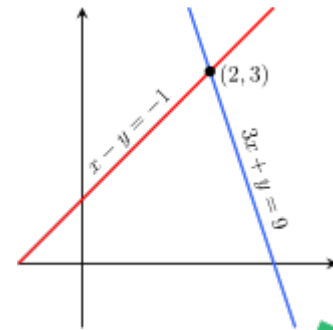
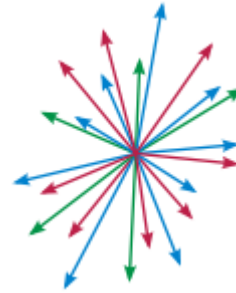
- Vectores

- Matrices

- Líneas

- Espacios vector $\begin{bmatrix} 2 & 4 & 7 & 3 \\ 3 & 4 & 0 & 1 \\ 0 & 5 & 2 & -1 \end{bmatrix}$

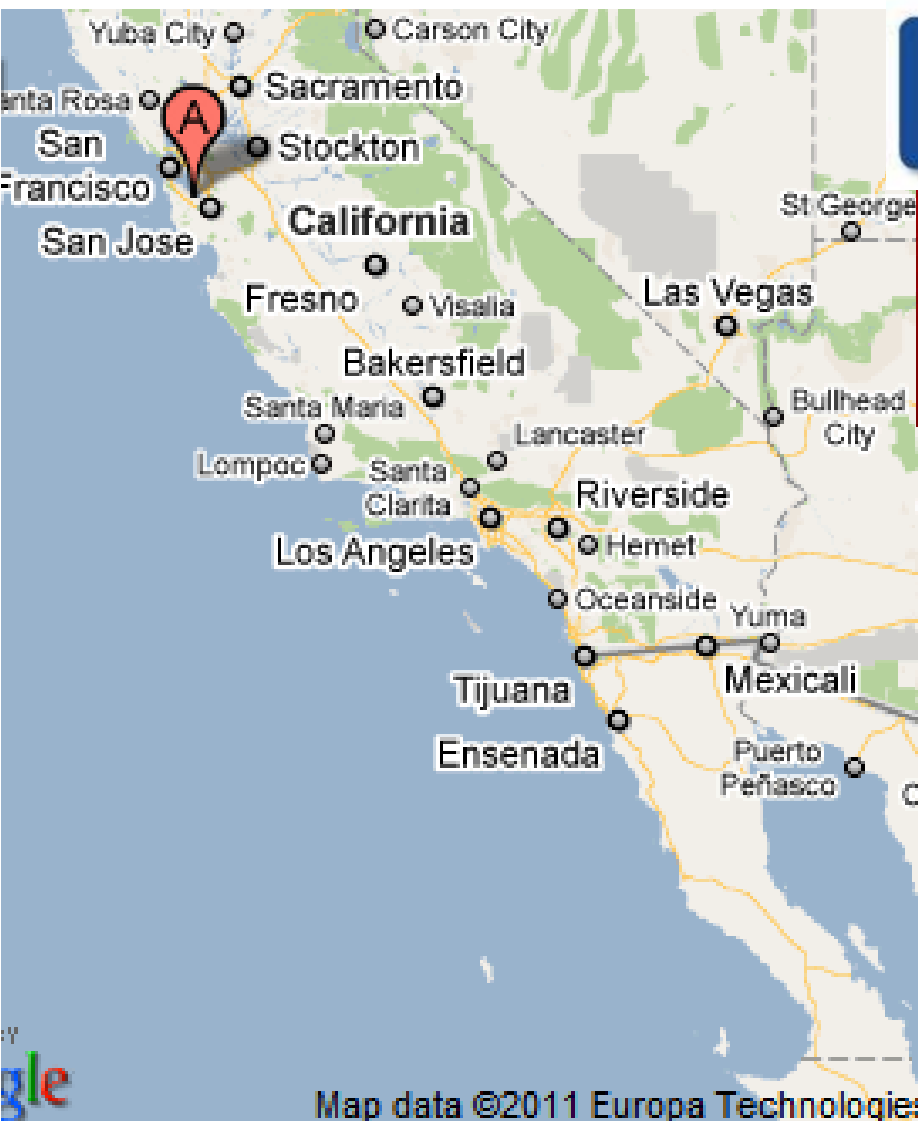
- Valores propios



$$\mathbf{A}\xi = \lambda\xi$$

¡A conquistar el mundo!





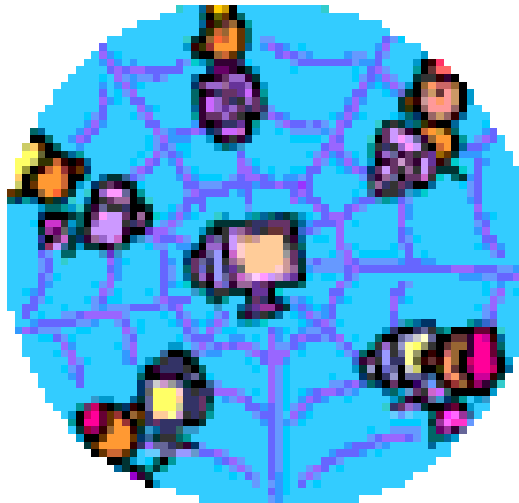
STANFORD
UNIVERSITY



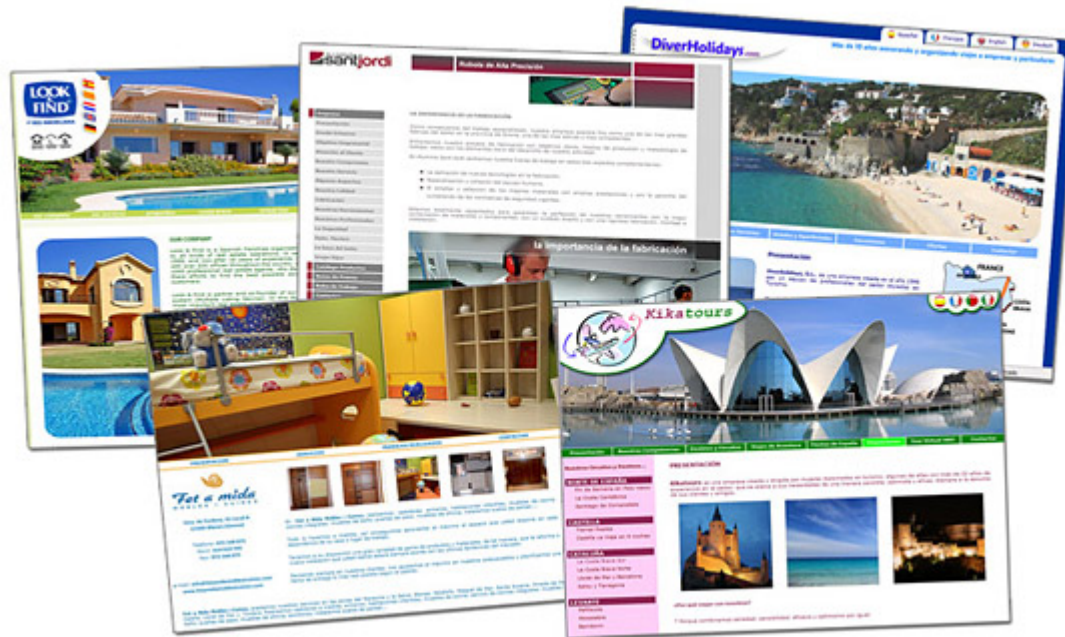
¿Qué es Internet?



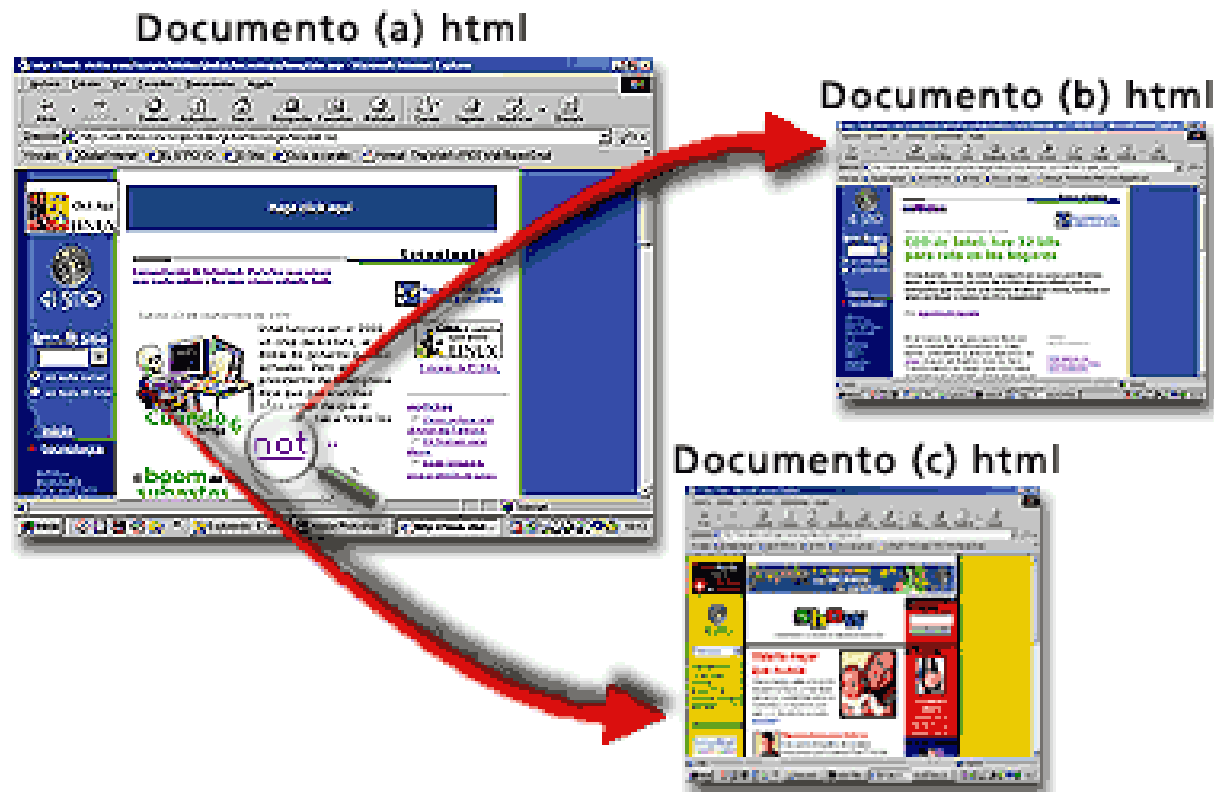
¿Qué es la Web?



Páginas Web



Enlaces, vínculos, hipervínculos



Términos

**Municipio
Tuxtla
Chiapas**

**Universidad
Chiapas
Ocelotes**

**Jaguares
Chiapas
Futbol**

**Ocelotes
Jaguares
Visita**

¿Cómo representar los datos?

Matrices

Matriz de términos

	Tuxtla	Universidad	Ocelotes	Visita	Jaguares	Municipio	Futbol	Chiapas
Ayuntamiento	0	0	0	0	0	0	0	0
UNACH	0	0	0	0	0	0	0	0
Deportes	0	0	0	0	0	0	0	0
ZOOMAT	0	0	0	0	0	0	0	0

Matriz de términos

Municipio
Tuxtla
Chiapas

	Tuxtla	Universidad	Ocelotes	Visita	Jaguares	Municipio	Futbol	Chiapas
Ayuntamiento	1	0	0	0	0	1	0	1
UNACH	0	0	0	0	0	0	0	0
Deportes	0	0	0	0	0	0	0	0
ZOOMAT	0	0	0	0	0	0	0	0

Matriz de términos

Universidad
Chiapas
Ocelotes

	Tuxtla	Universidad	Ocelotes	Visita	Jaguares	Municipio	Futbol	Chiapas
Ayuntamiento	1	0	0	0	0	1	0	1
UNACH	0	1	1	0	0	0	0	1
Deportes	0	0	0	0	0	0	0	0
ZOOMAT	0	0	0	0	0	0	0	0

Matriz de términos

	Tuxtla	Universidad	Ocelotes	Visita	Jaguares	Municipio	Futbol	Chiapas
Ayuntamiento	1	0	0	0	0	1	0	1
UNACH	0	1	1	0	0	0	0	1
Deportes	0	0	0	0	1	0	1	1
ZOOMAT	0	0	1	1	1	0	0	0

¿Y ahora?



- Supongamos que queremos buscar

Universidad de Chiapas

Tuxtla	Universidad	Ocelotes	Visita	Jaguares	Municipio	Futbol	Chiapas
0	1	0	0	0	0	0	1

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

1x0

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{pmatrix}$$

$1 \times 0 + 0 \times 1$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{pmatrix}$$

$$1 \times 0 + 0 \times 1 + 0 \times 0$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{pmatrix}$$

$$1 \times 0 + 0 \times 1 + 0 \times 0 + 0 \times 0$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{pmatrix}$$

$$\begin{aligned}
 &1 \times 0 + 0 \times 1 + 0 \times 0 + 0 \times 0 + 0 \times 0 \\
 &\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \end{pmatrix}
 \end{aligned}$$

$$1 \times 0 + 0 \times 1 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 0 = ?$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \end{pmatrix}$$

$$\begin{aligned}
 &1 \times 0 + 0 \times 1 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 0 + 0 \times 0 + 0 \times 0 \\
 &\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{pmatrix}
 \end{aligned}$$

The image shows a matrix multiplication problem. The first row of the matrix is $(1, 0, 0, 0, 0, 1, 0, 1)$ and the first row of the vector is $(0, 1, 0, 0, 0, 0, 0, 1)$. The result of the first row multiplication is $1 \times 0 + 0 \times 1 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 0 + 0 \times 0 + 0 \times 0$. The result is 0 . The second row of the matrix is $(0, 1, 1, 0, 0, 0, 0, 1)$ and the second row of the vector is $(0, 1, 0, 0, 0, 0, 0, 1)$. The result of the second row multiplication is $0 \times 0 + 1 \times 1 + 1 \times 0 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 1$. The result is 2 . The third row of the matrix is $(0, 0, 0, 0, 1, 0, 1, 1)$ and the third row of the vector is $(0, 1, 0, 0, 0, 0, 0, 1)$. The result of the third row multiplication is $0 \times 0 + 0 \times 1 + 0 \times 0 + 0 \times 0 + 1 \times 0 + 0 \times 0 + 1 \times 0 + 1 \times 1$. The result is 1 . The fourth row of the matrix is $(0, 0, 1, 1, 1, 0, 0, 0)$ and the fourth row of the vector is $(0, 1, 0, 0, 0, 0, 0, 1)$. The result of the fourth row multiplication is $0 \times 0 + 0 \times 1 + 1 \times 0 + 1 \times 0 + 1 \times 0 + 0 \times 0 + 0 \times 0 + 0 \times 1$. The result is 0 . The result vector is $(0, 2, 1, 0, \dots, \dots, \dots, \dots)$.

$$1 \times 0 + 0 \times 1 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 0 + 0 \times 0 + 1 \times 1$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} ? \\ - \\ - \\ - \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

Matriz de términos

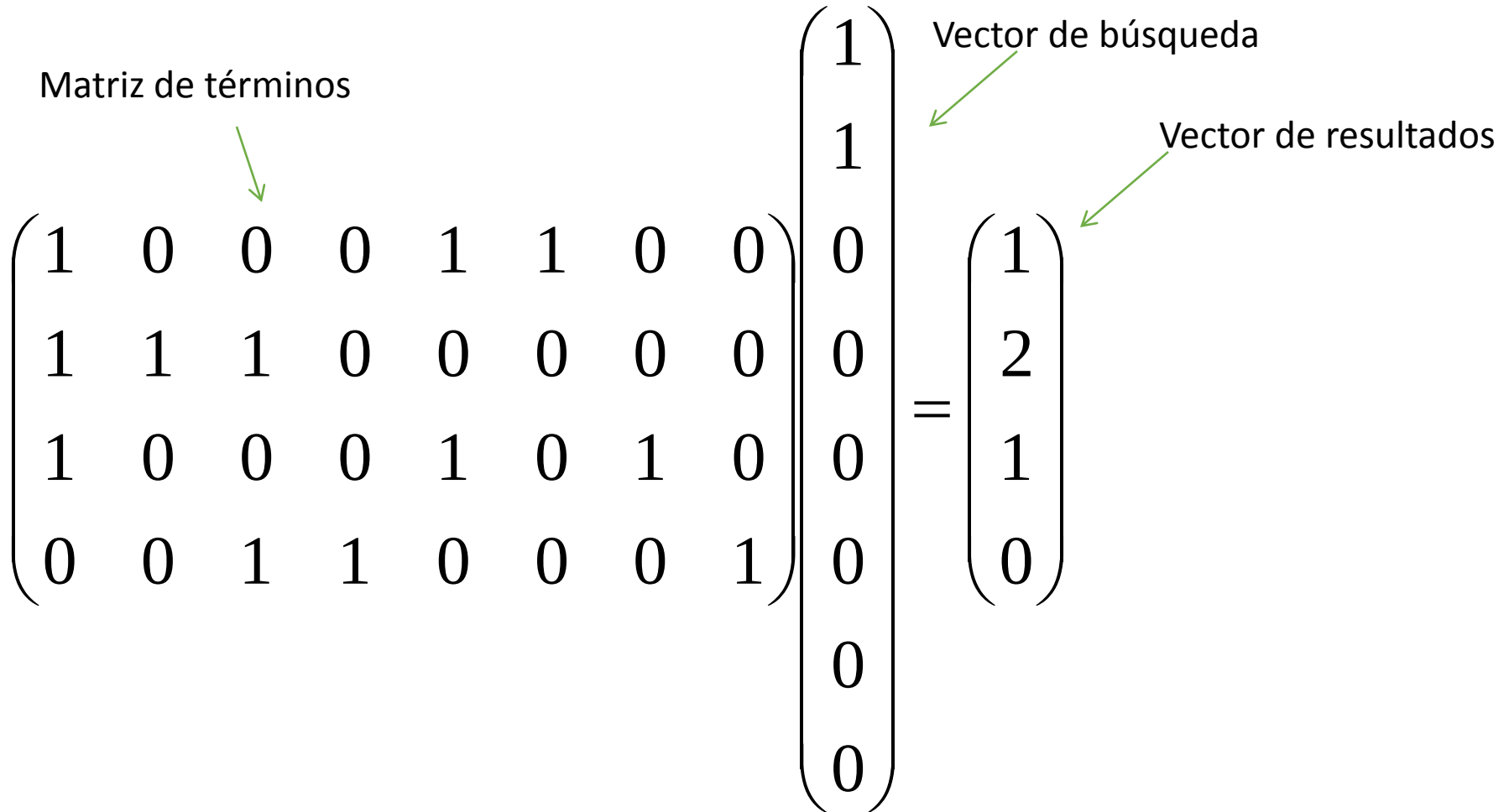


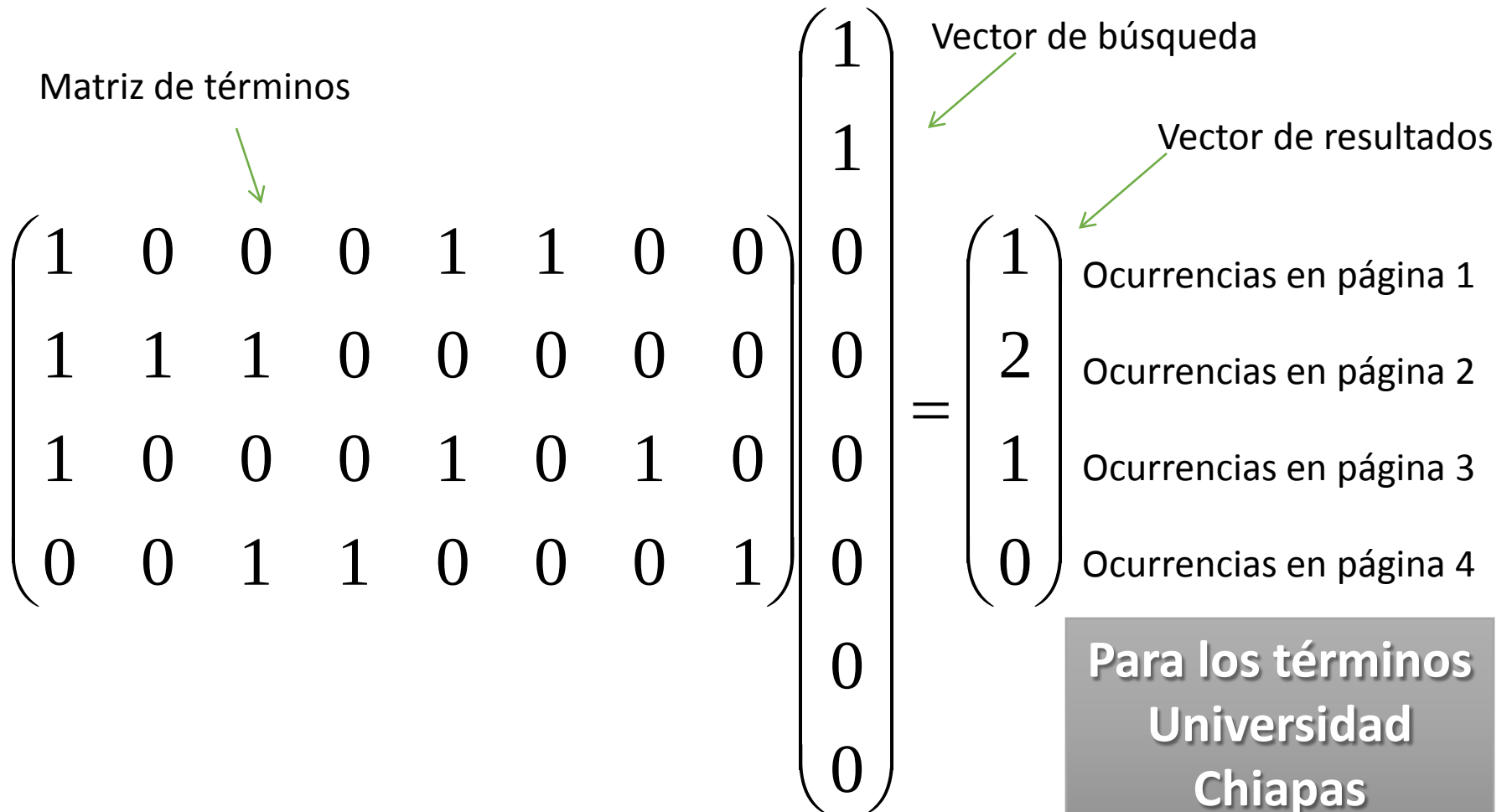
Diagram illustrating a matrix multiplication operation:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

Labels and arrows:

- Matriz de términos (points to the first matrix)
- Vector de búsqueda (points to the second matrix)
- Vector de resultados (points to the third matrix)

Matriz de términos



The diagram illustrates a matrix multiplication operation. On the left is a 4x8 matrix labeled 'Matriz de términos'. A green arrow points to the third column of this matrix. To its right is a vertical vector labeled 'Vector de búsqueda' with a green arrow pointing to its top element. These are multiplied to produce a vertical vector labeled 'Vector de resultados' with a green arrow pointing to its top element. The result vector has four elements corresponding to the rows of the matrix. To the right of the result vector are four labels: 'Ocurrencias en página 1', 'Ocurrencias en página 2', 'Ocurrencias en página 3', and 'Ocurrencias en página 4'.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

Vector de búsqueda

Vector de resultados

Ocurrencias en página 1

Ocurrencias en página 2

Ocurrencias en página 3

Ocurrencias en página 4

Para los términos
Universidad
Chiapas

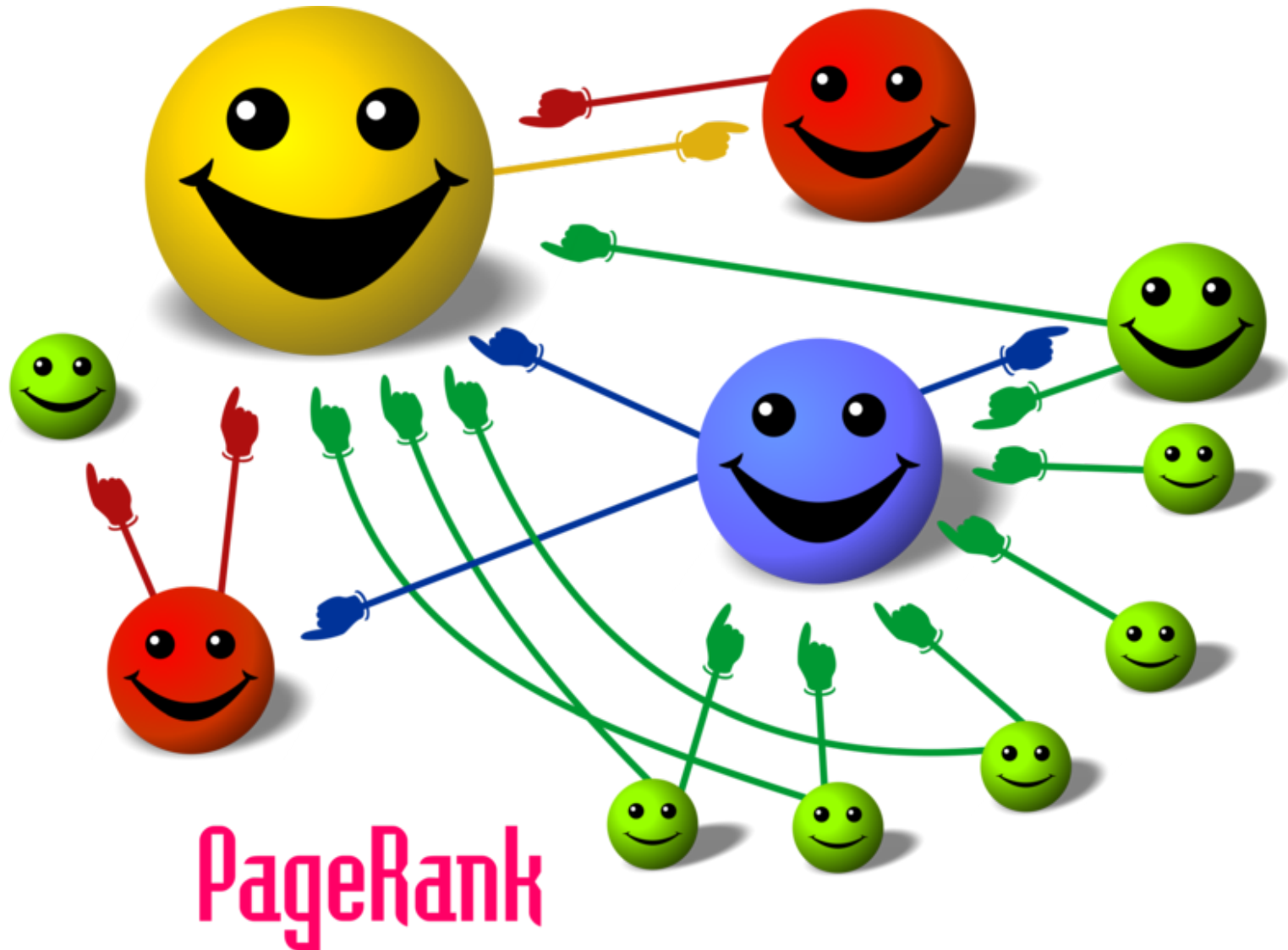
¿En qué
orden los
escribo?



Search and Display the Results

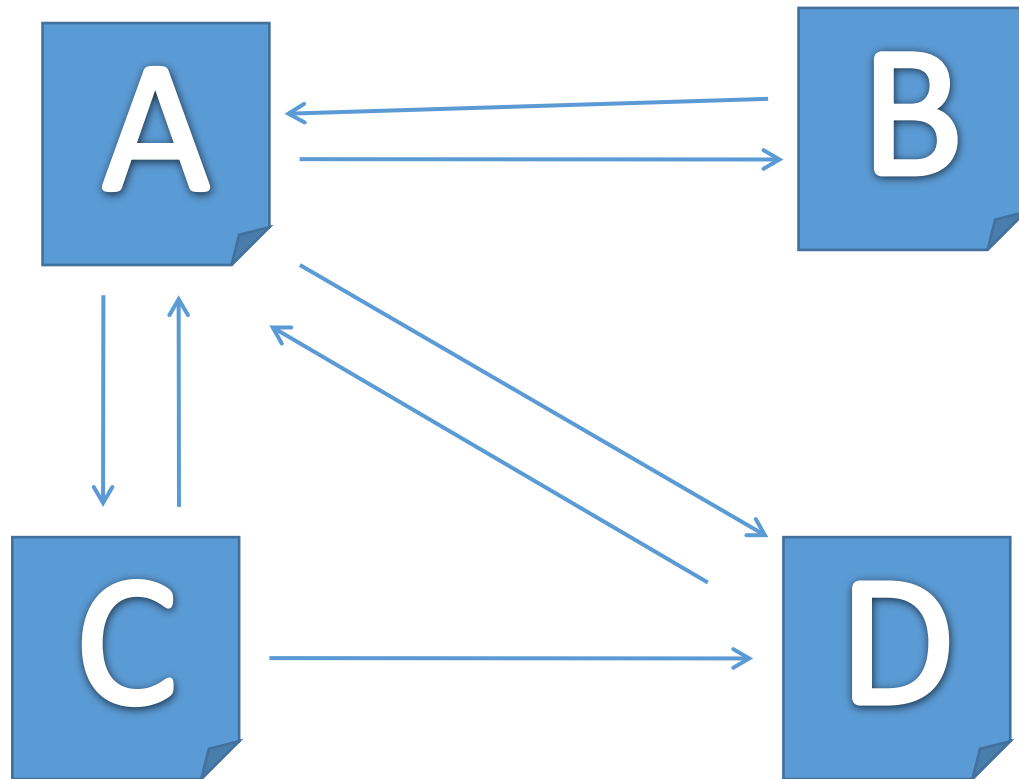
To find an old friend or information about someone, try: **Santa Claus** or **"Santa Claus"**
Using a capital letter ensures that only that capitalization will be found.

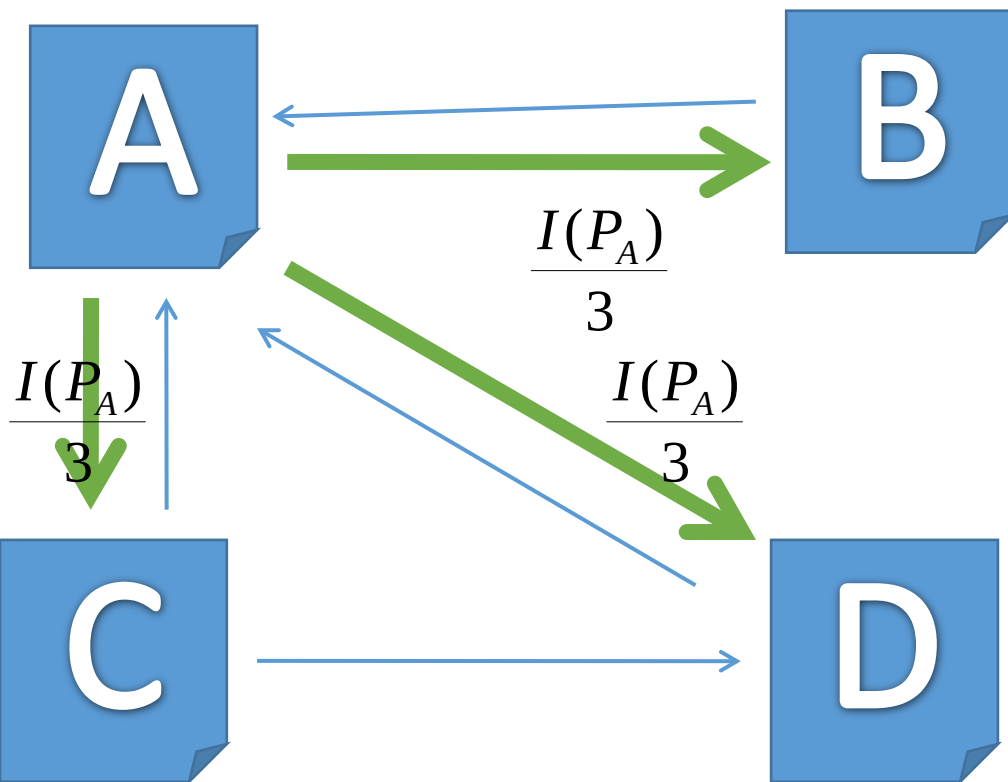
PageRank



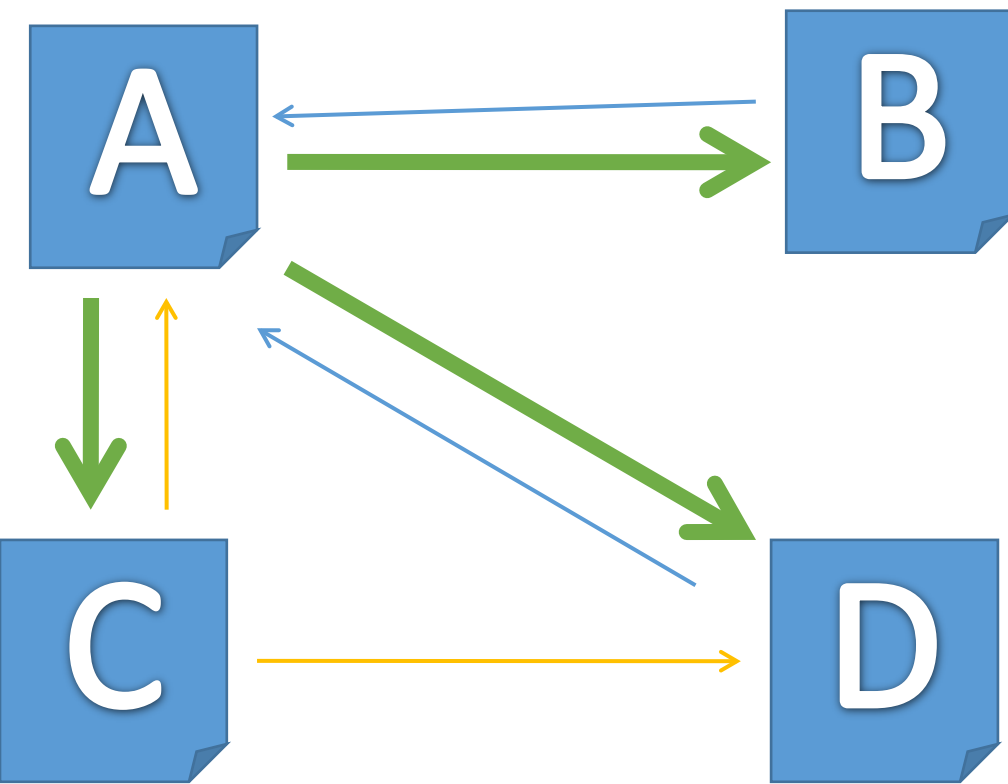
Enlaces

Links, Hyperlinks, Vínculos, Hipervínculos

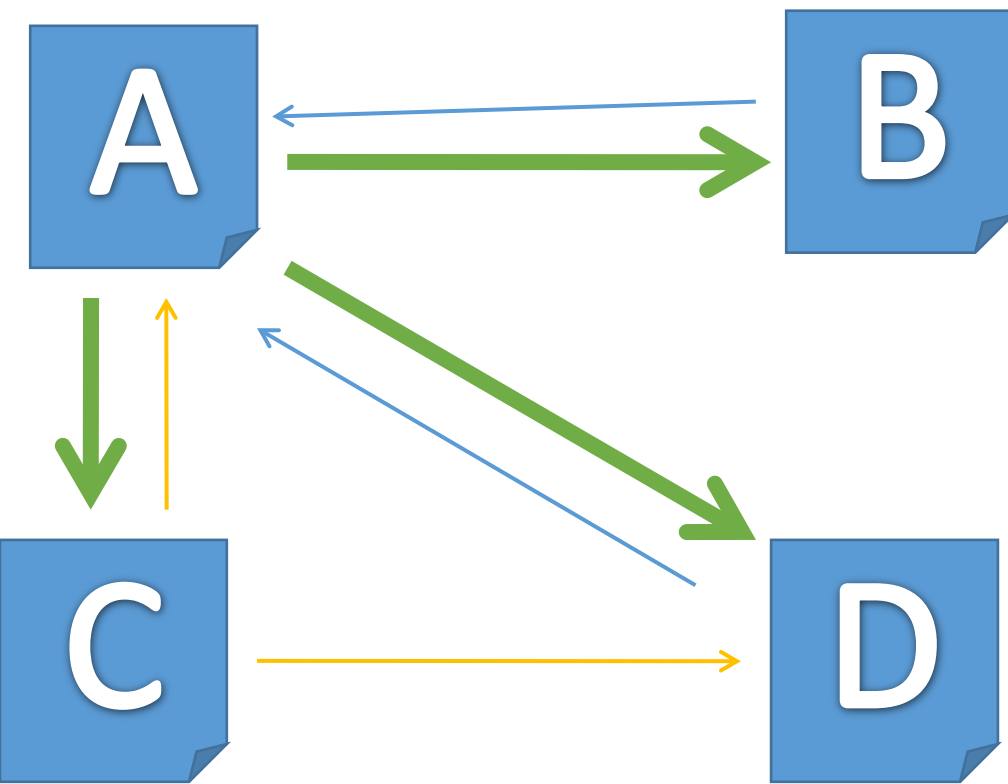




$$I(P_A)$$



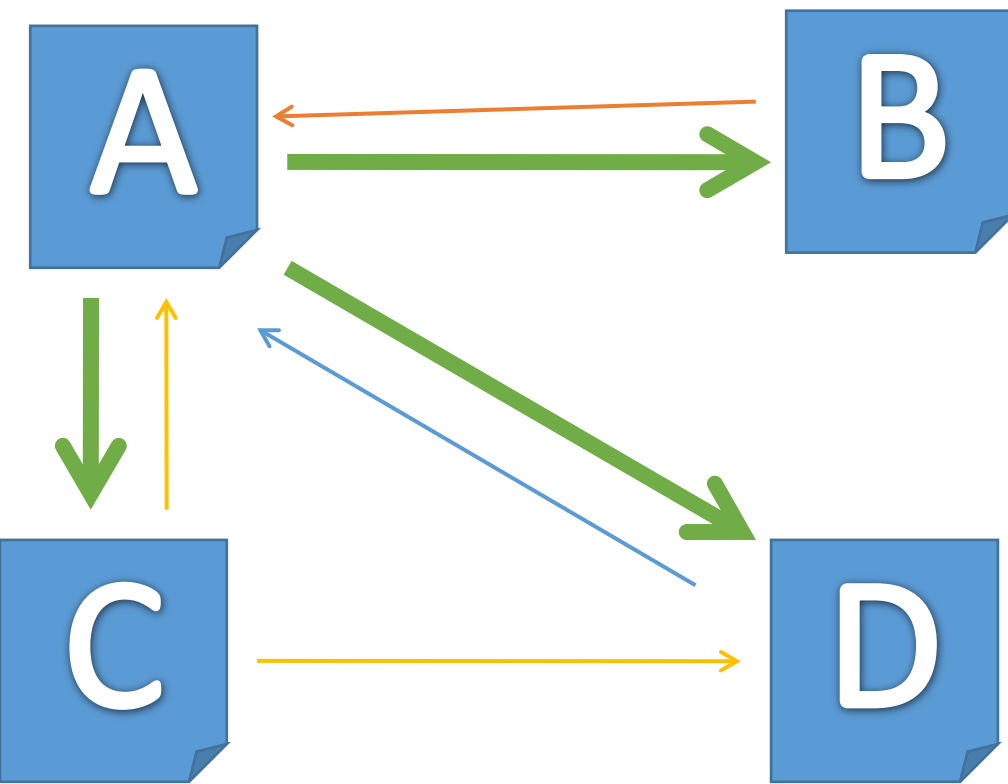
$$I(P_D) = \frac{I(P_A)}{3} + \frac{I(P_C)}{2}$$



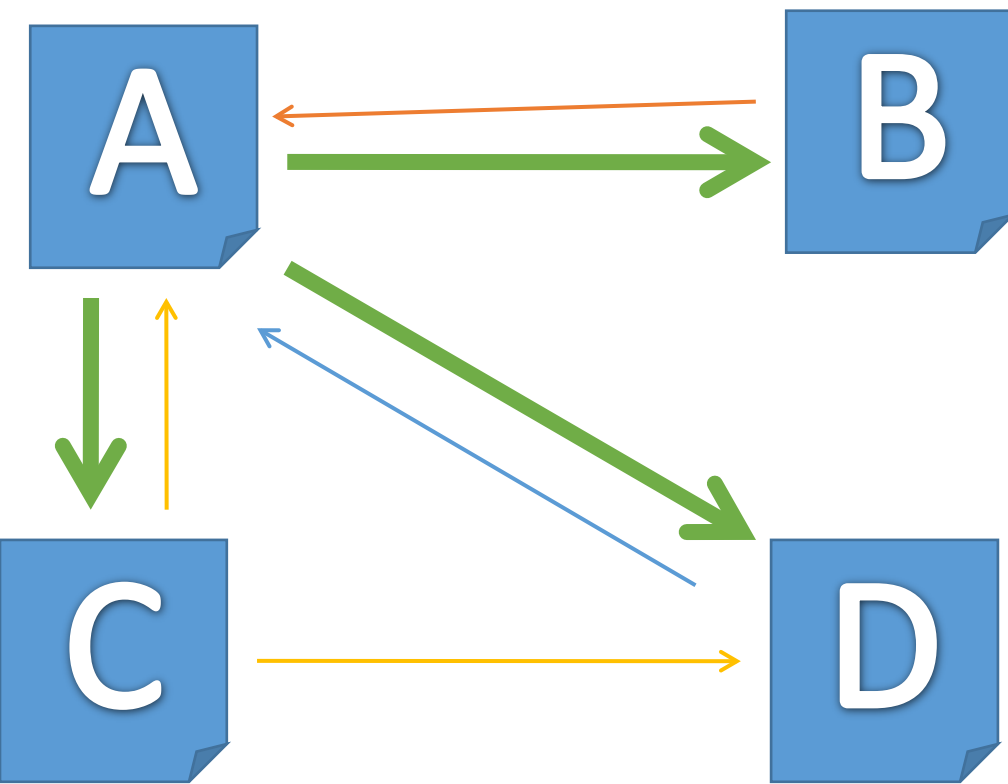
$$I(P_B) = \frac{I(P_A)}{3}$$

$$I(P_D) = \frac{I(P_A)}{3} + \frac{I(P_C)}{2}$$

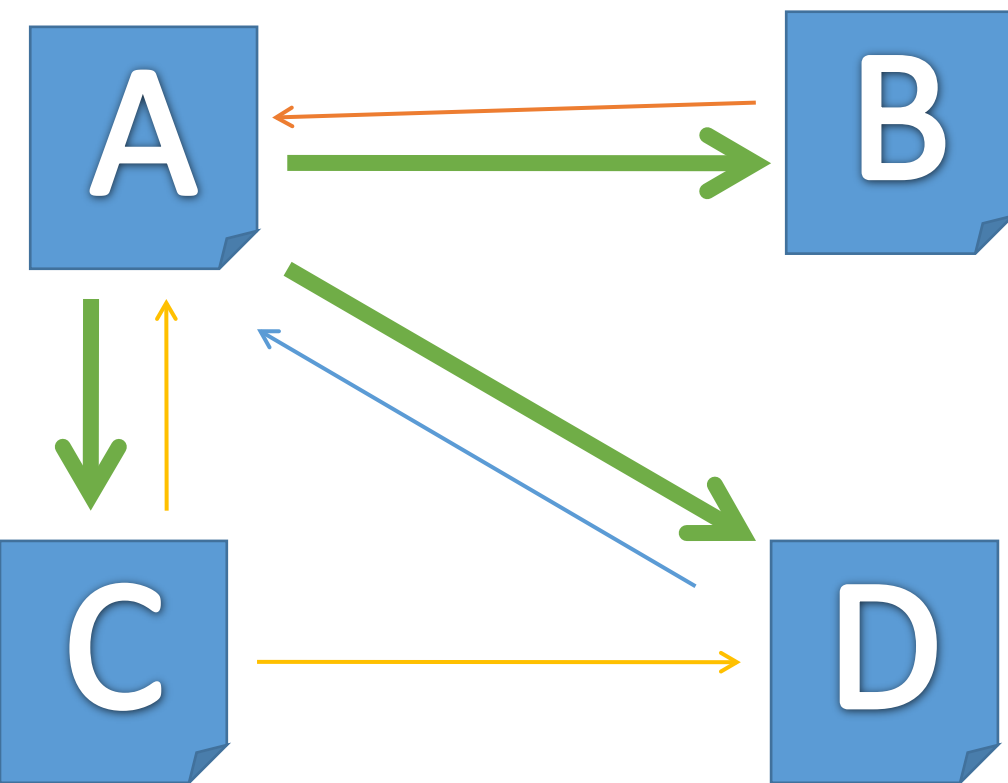
Matrices



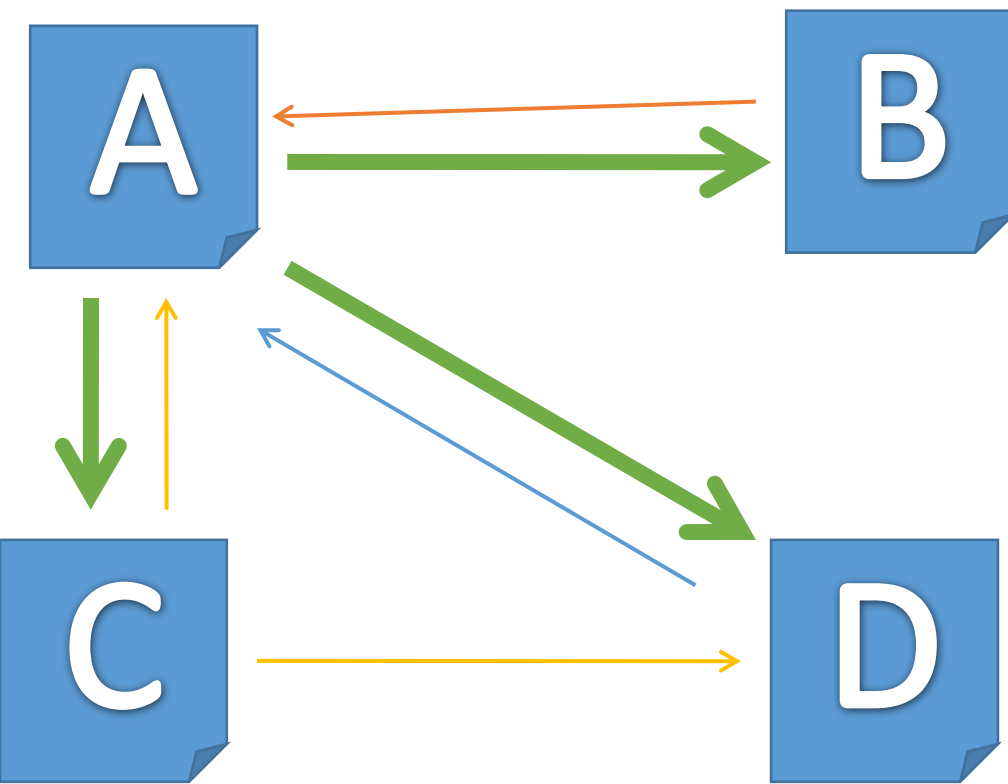
$$\begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{pmatrix} A & B & C & D \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



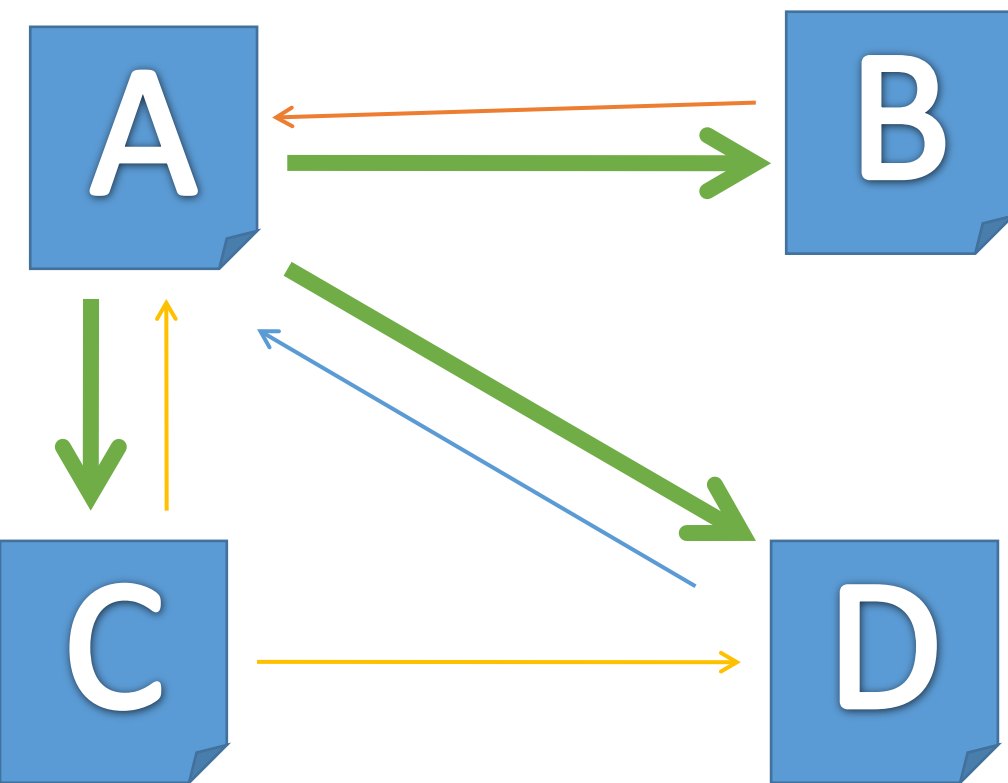
$$\begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{pmatrix} A & B & C & D \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$



$$\begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$\begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{pmatrix} A & B & C & D \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \end{pmatrix}$$



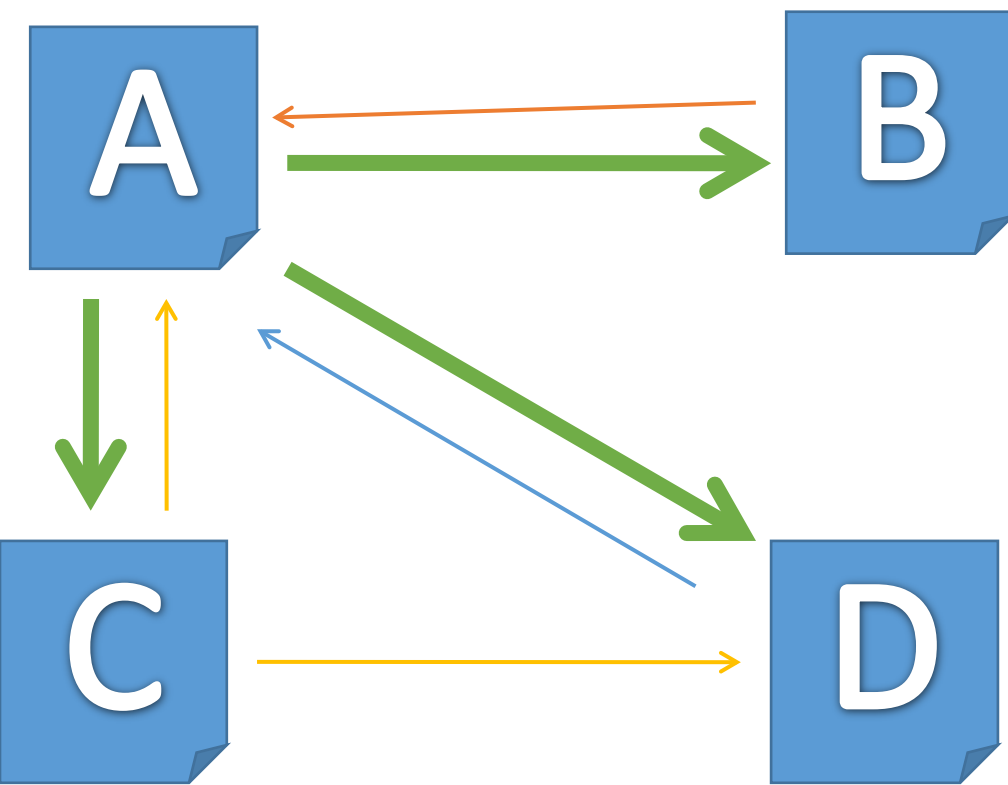
A

B

C

D

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

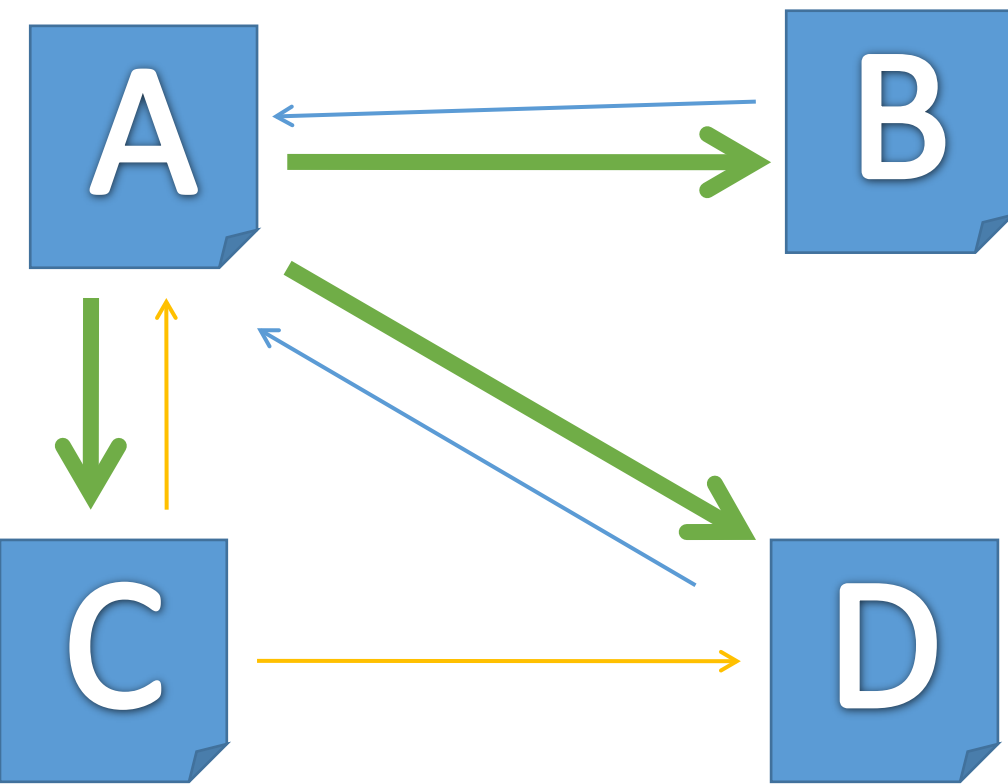


$$\begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 0 & 1 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \end{pmatrix} \end{matrix}$$

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix} = \begin{pmatrix} \frac{I(P_B)}{1} + \frac{I(P_C)}{2} + \frac{I(P_D)}{1} \\ \frac{I(P_A)}{3} \\ \frac{I(P_A)}{3} \\ \frac{I(P_A)}{3} + \frac{I(P_C)}{2} \end{pmatrix}$$



$$I(P_B) = \frac{I(P_A)}{3}$$

$$I(P_D) = \frac{I(P_A)}{3} + \frac{I(P_C)}{2}$$

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix} = \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix}$$

Valores propios y vectores propios

Matriz

Valor propio

$$Ax = \lambda x$$

Vector propio

$$A\chi = \lambda\chi$$

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix} = \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix}$$

$$A\chi = \lambda\chi$$

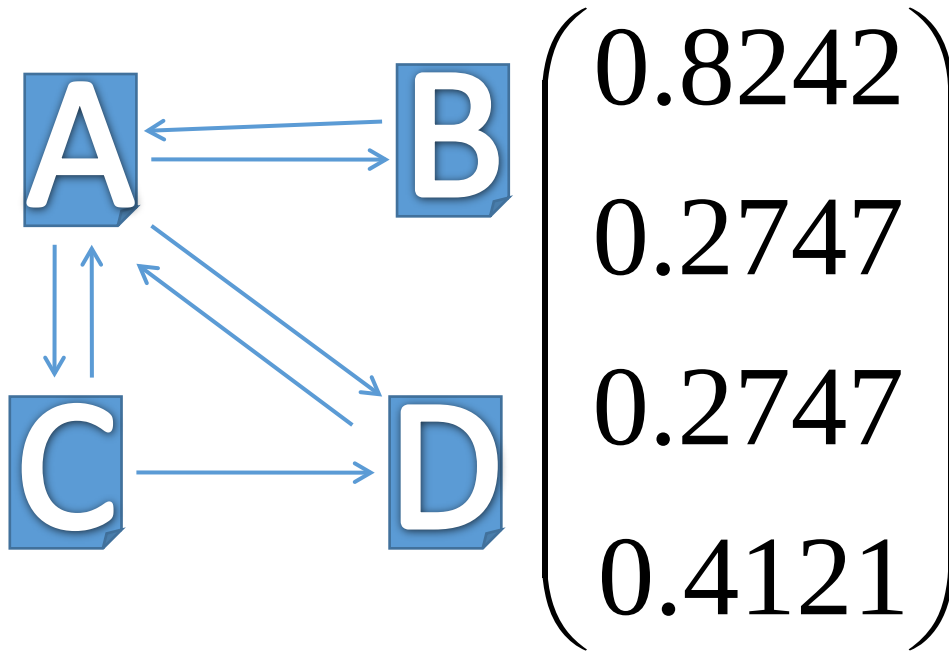
$$A \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix} = \begin{pmatrix} I(P_A) \\ I(P_B) \\ I(P_C) \\ I(P_D) \end{pmatrix}$$

$$Ax = \lambda x$$

$$Ax = x$$

MATLAB

Vector de PageRank



Página A

Página B

Página C

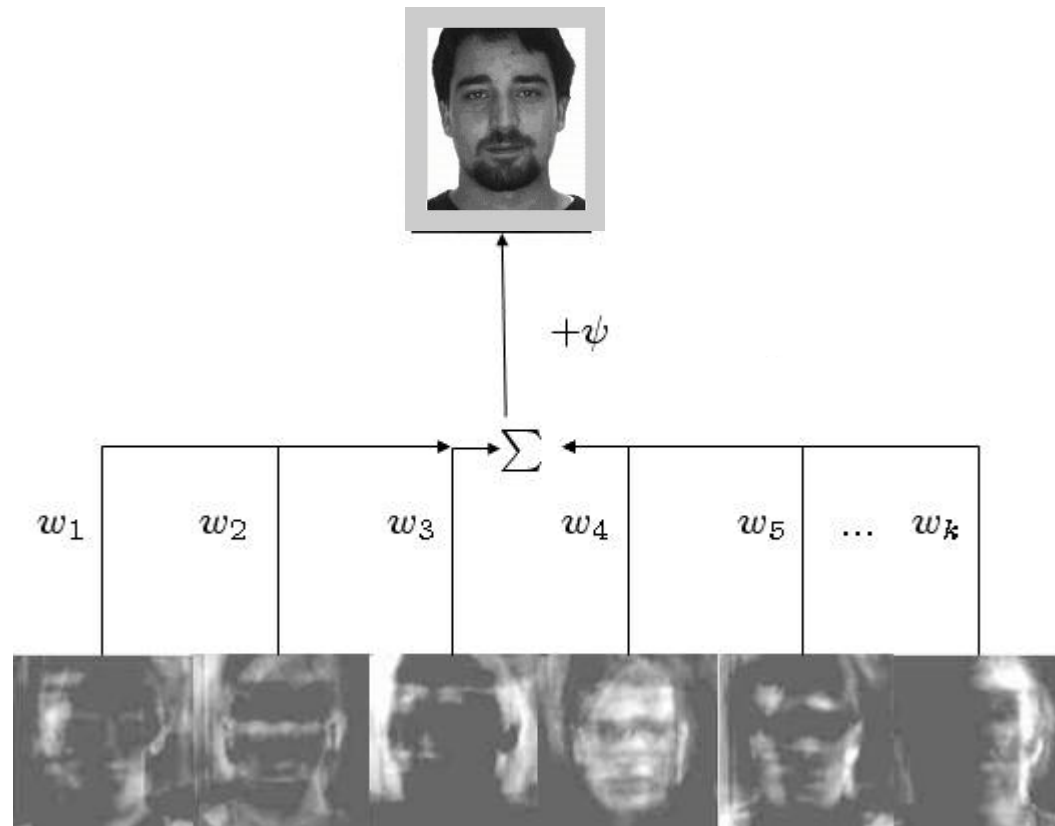
Página D

Bueno, bueno, pero
¿y qué hay de las fotos?



Eigenfaces





$$S = \{ \Gamma_1, \Gamma_2, \Gamma_3, \dots, \Gamma_M \}$$

$$\Psi = \frac{1}{M} \sum_{n=1}^M \Gamma_n$$



$$\Phi_i = \Gamma_i - \Psi$$

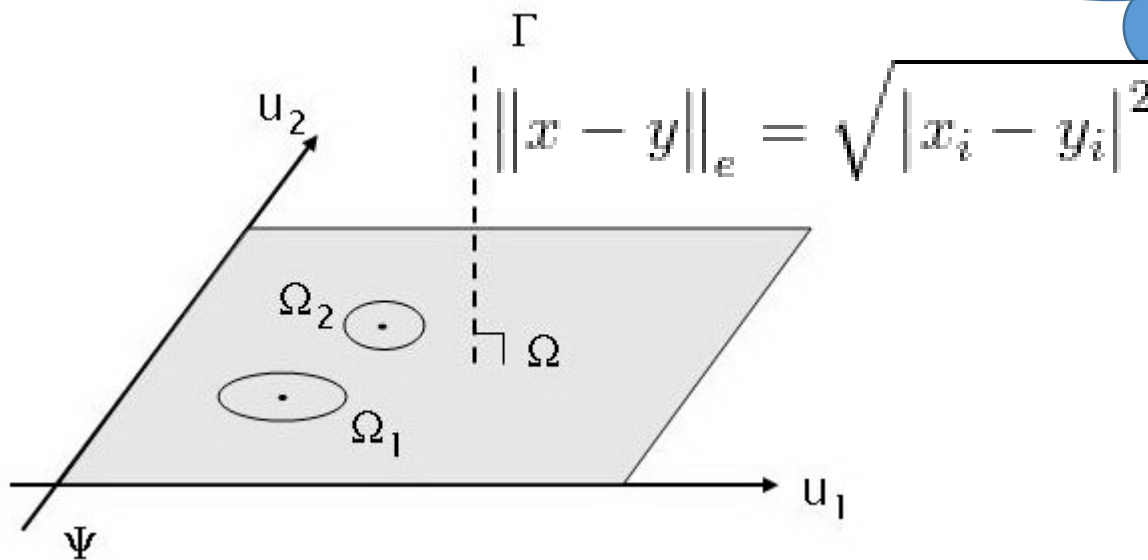
$$C = AA^T \quad A = [\Phi_1, \Phi_2 \dots \Phi_M]$$



Sí, sí, pero yo quiero saber
dónde está este señor...

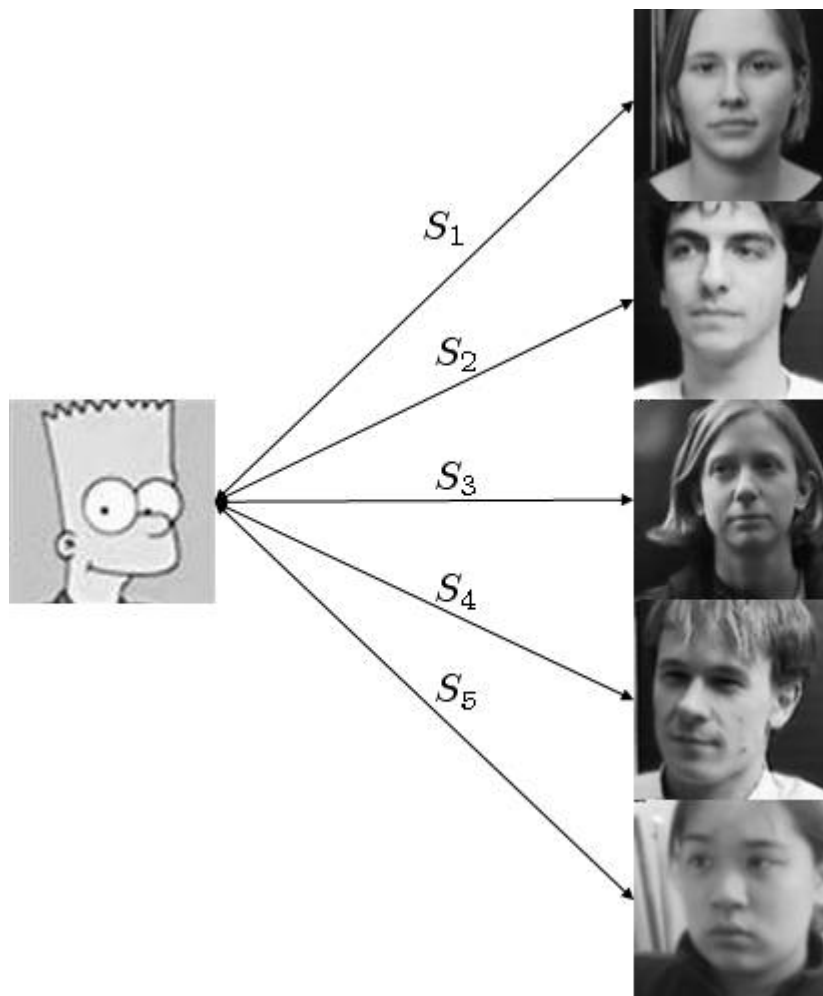


El espacio de caras

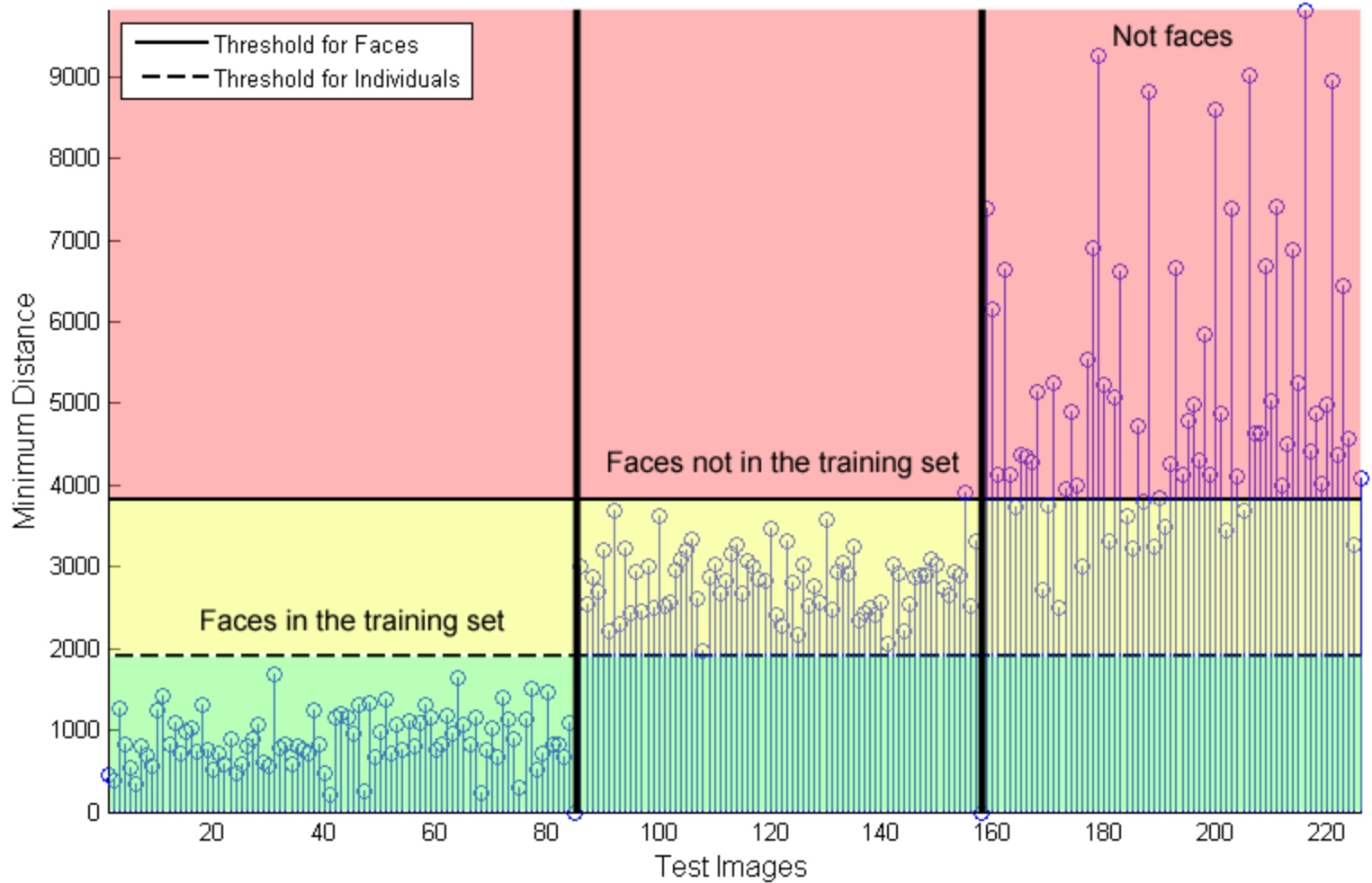


Distancia





Threshold Calculation





$$Ax = y$$

$$Ax = y$$

Si A es invertible,

$$x = A^{-1} y$$

Si A no es invertible,

Sea

$$A_{p \times q} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

De forma que A_{11} es una matriz de orden $r \times r$ y rango r

Tomemos

$$G_{q \times p} = \begin{bmatrix} A_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix}$$

Inversa Moore-Penrose

- $AGA = A$
- $GAG = G$
- $(AG)^t = AG$
- $(GA)^t = GA$

Es única.

La solución general del sistema de ecuaciones

$$Ax = b$$

Está dado por

$$x = Gb + (I - GA)z$$

Donde z es arbitrario.

Gracias