“EL UNIVERSO OBSCURO”

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Contenido

1. - Modelos cosmológicos de Friedmann-Robertson-Walker (FRW).

2. - El Diagrama de Hubble: distancia modular vs corrimiento al rojo.

3.- La distancia de luminosidad y la relación distancia modular vs corrimiento al rojo.

4.- Estimación Bayesiana de parámetros.

5.- Ecuaciones cosmológicas de la Teoría General de la Relatividad.

6.- Modelo del Universo compuesto por Materia Bariónica, Materia Obscura, Radiación de fotones del CMB y Constante Cosmológica.

7.- Evidencia para la existencia de la Constante Cosmológica.

1. Modelos cosmológicos de Friedmann-Robertson-Walker (FRW)

Principio Cosmológico: El universo es espacialmente homogéneo e isotrópico en escalas de distancia cosmológicas (Einstein, 1917) pero evoluciona con el tiempo.

Este Principio engloba la idea: los humanos no somos observadores privilegeados y no estamos en el centro de nuestro universo.

- **Evidencia observacional para isotropía espacial en escalas cosmológicas:**

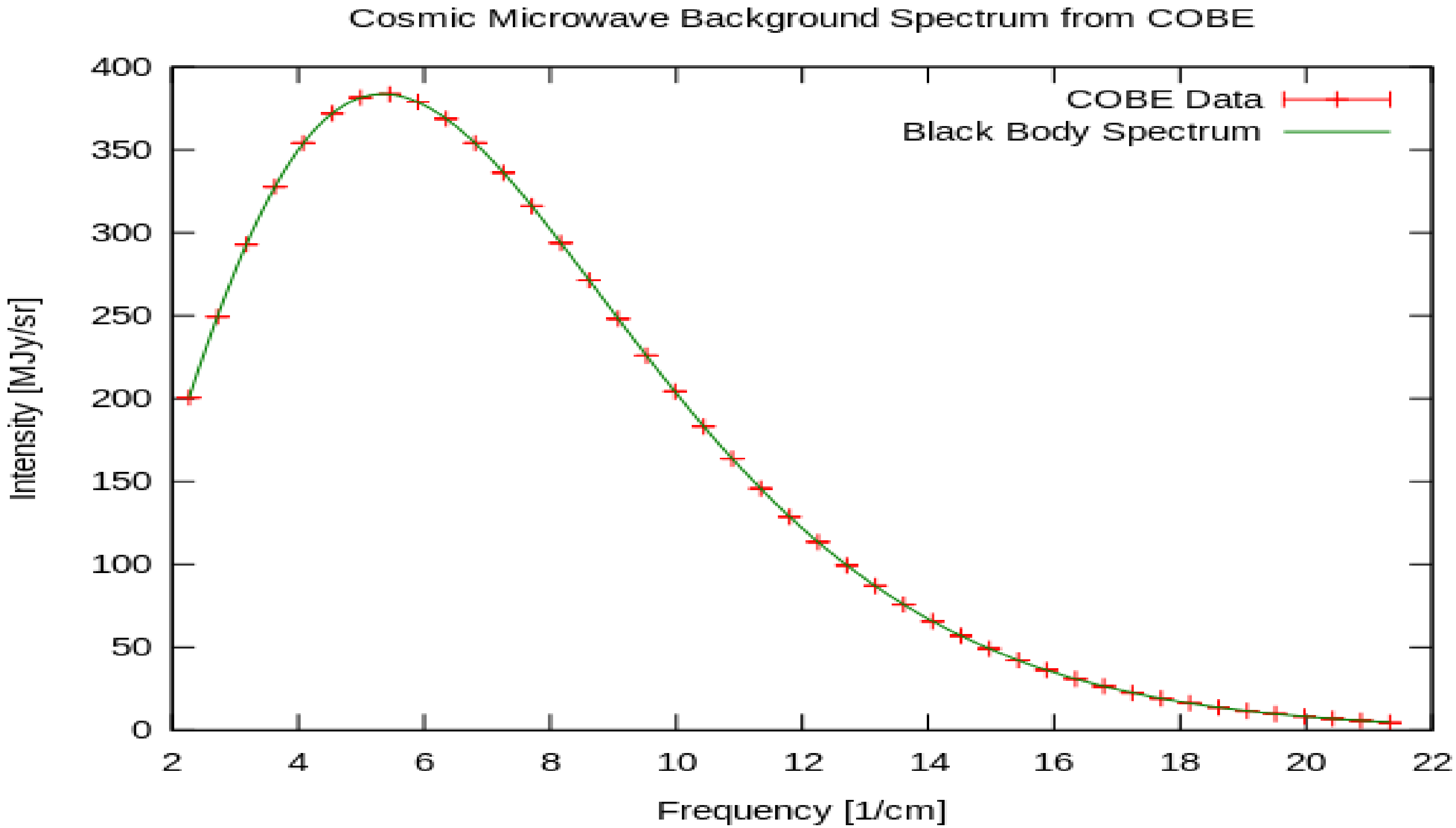
- Medidas de la Radiación Cósmica del Fondo de Microondas (CMB) con temperatura promedio: T = 2.728 K.

-Experimentos:
-Cosmic Background Explorer (COBE) (1992).
-Wilkinson Microwave Anisotropy Probe (WMAP-9) (2003-2011).
-Planck Satellite (2015).

- Fotones que han viajado hasta hoy desde el tiempo (14 Giga años) en que se desacoplan del plasma de electrones y protones una vez que éstos se recombinan para formar los elementos más ligeros (hidrógeno, helio, litio) a la temperatura T = 3000 K (z=1100).

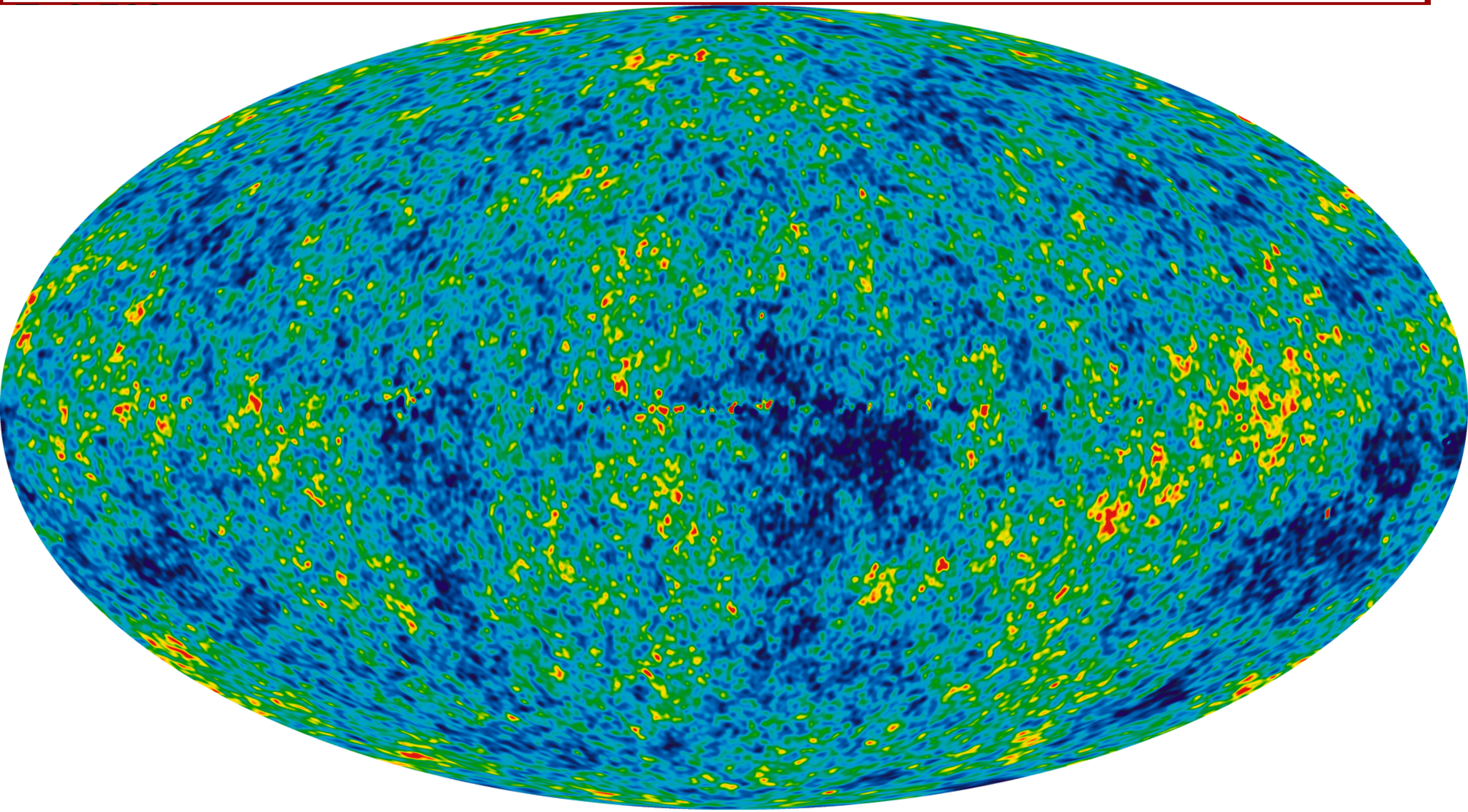
Gas de Radiación Cósmica de Fotones (Distribución de Cuerpo Negro)
Temperatura media *T* = 2.728 Kelvin.

$$\rho_r^0 = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4 = 4.7 \times 10^{-34} \frac{gr}{cm^3}$$



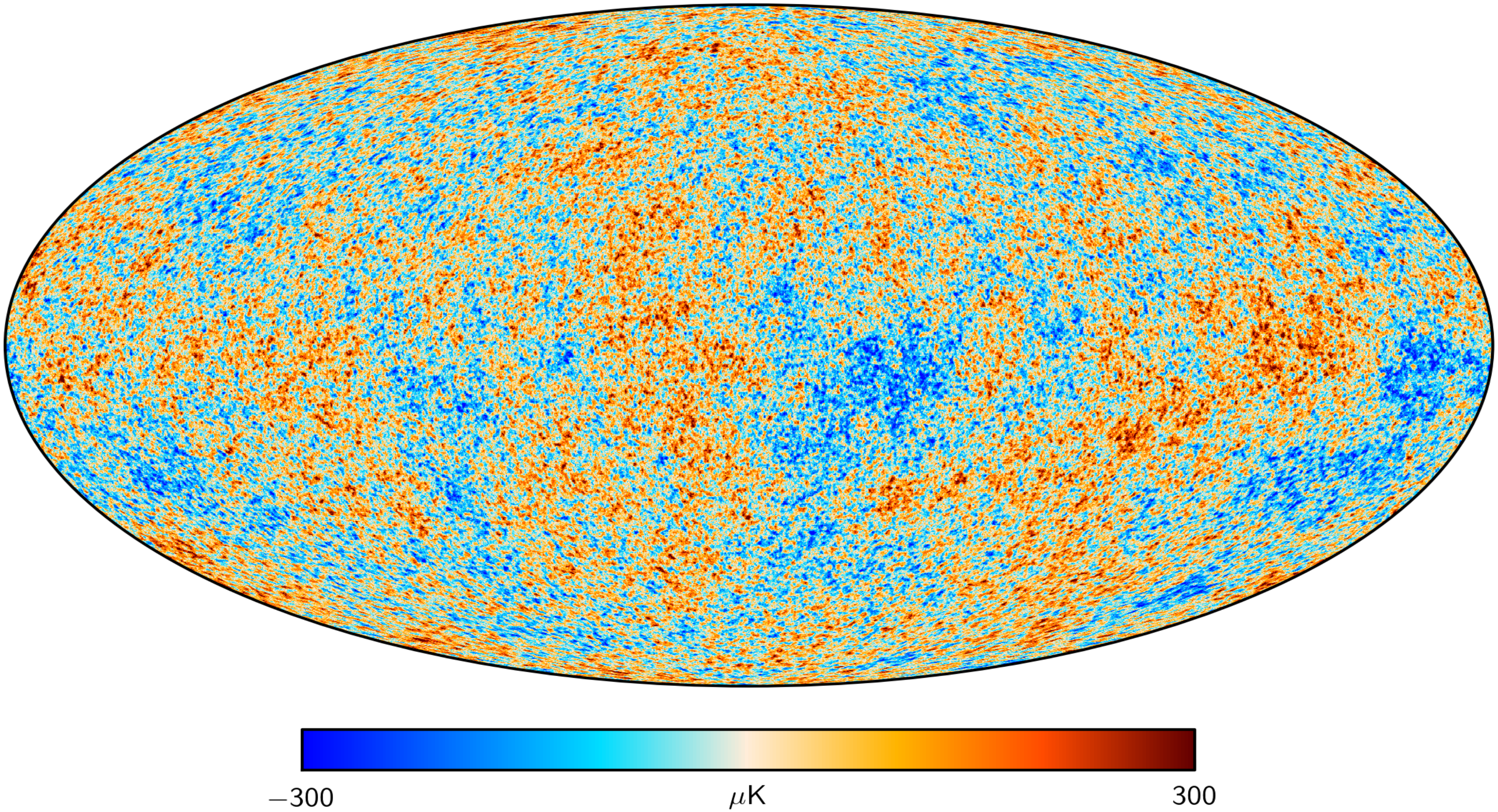
Anisotropías de la radiación cósmica medidas por el experimento Wilkinson Microwave Anisotropy Probe (WMAP-7). E. Komatsu et al., The Astrophysical Journal Supplement Series, 192:18 (2011).

Esta imagen corresponde a variaciones ΔT = (-200, 200) Microkelvin de la temperatura media T= 2.728 Kelvin.



Anisotropías de la radiación cósmica medidas por el experimento PLANCK (2015). http://www.cosmos.esa.int/web/planck/publications#Planck2015

Esta imagen corresponde a variaciones ΔT = (-300, 300) Microkelvin de la temperatura media T= 2.728 Kelvin.



Principio Cosmológico: El universo es espacialmente homogéneo e isotrópico en escalas de distancia cosmológicas (Einstein, 1917) pero evoluciona con el tiempo.

Este Principio engloba la idea: los humanos no somos observadores privilegeados y no estamos en el centro de nuestro universo.

- **Evidencia parcial para homogeneidad espacial proviene de medidas de distribución de galaxias por los experimentos:**

(1) Two Degree Field Galaxy Redshift Survey (2dF) con 250,000 galaxias (2001- 2003).

(2) Sloan Digital Sky Survey (SDSS-III) con 893,000 galaxias sobre 9100 grados cuadrados (2000 – 2014).

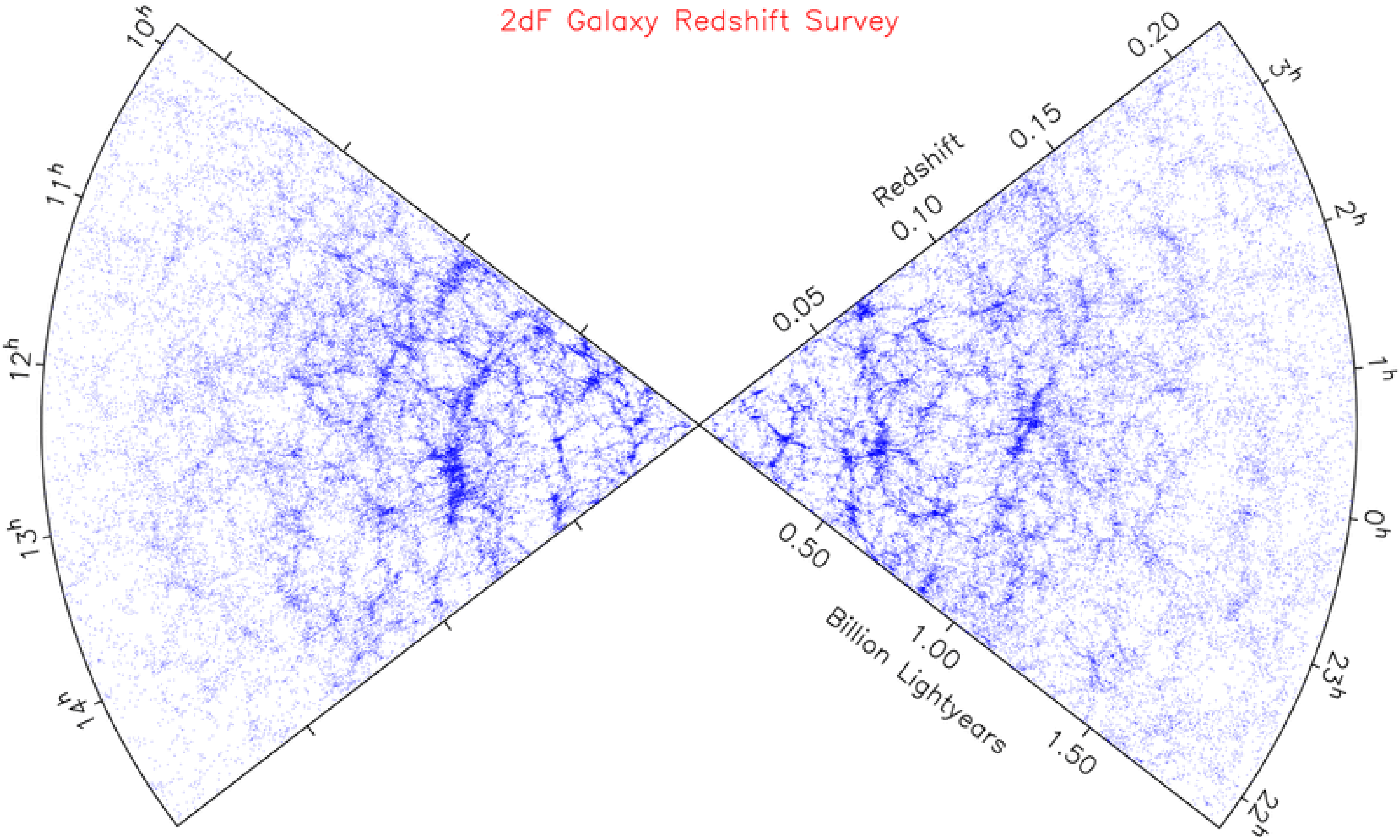
Experimentos futuros:

(3) Sloan Digital Sky Survey (SDSS-IV) (2014-2020):

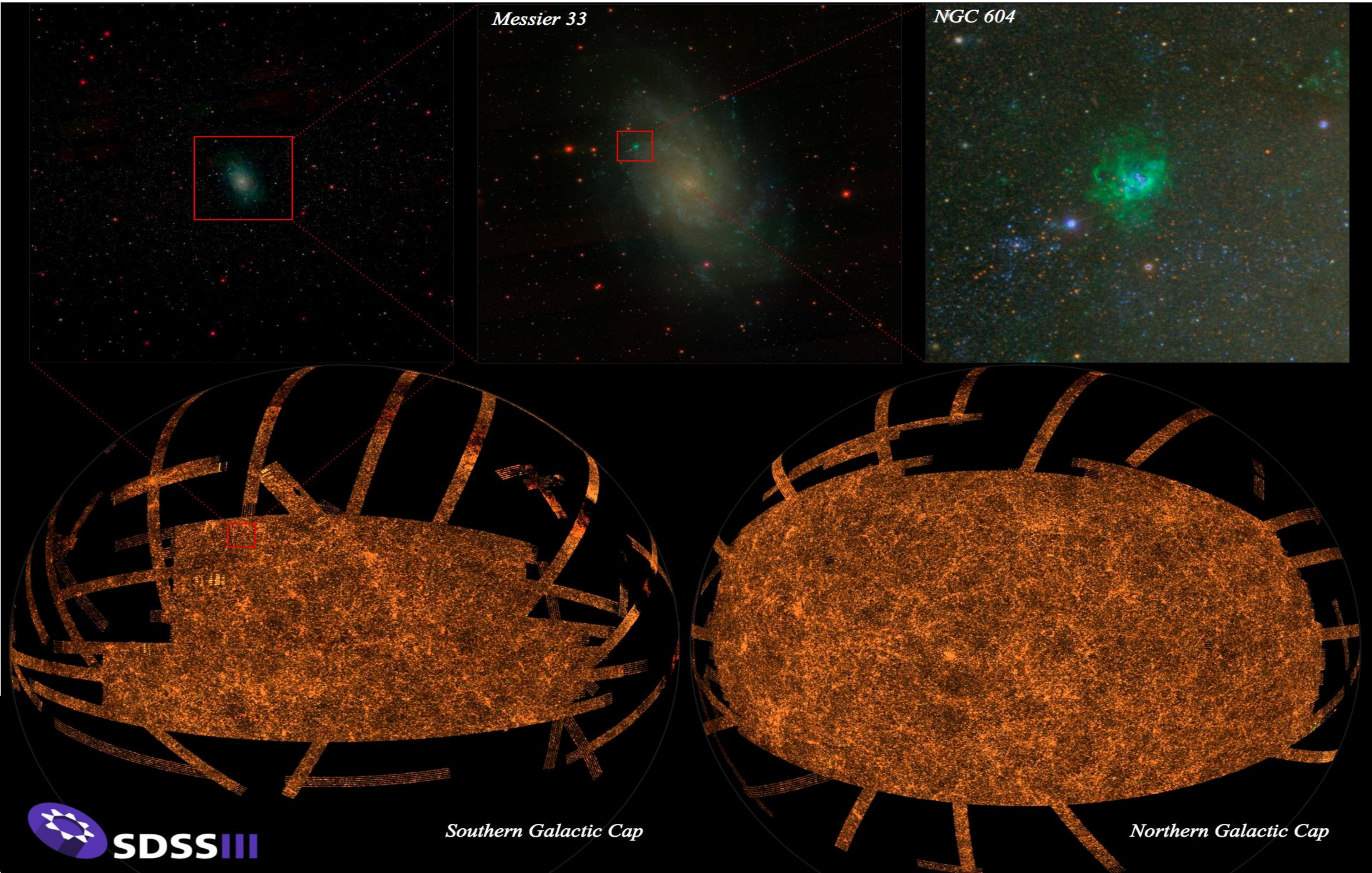
(Extended) Baryon Oscillation Spectroscopic Survey (BOSS, eBOSS)

Campo profundo observado por el telescopio espacial Hubble.

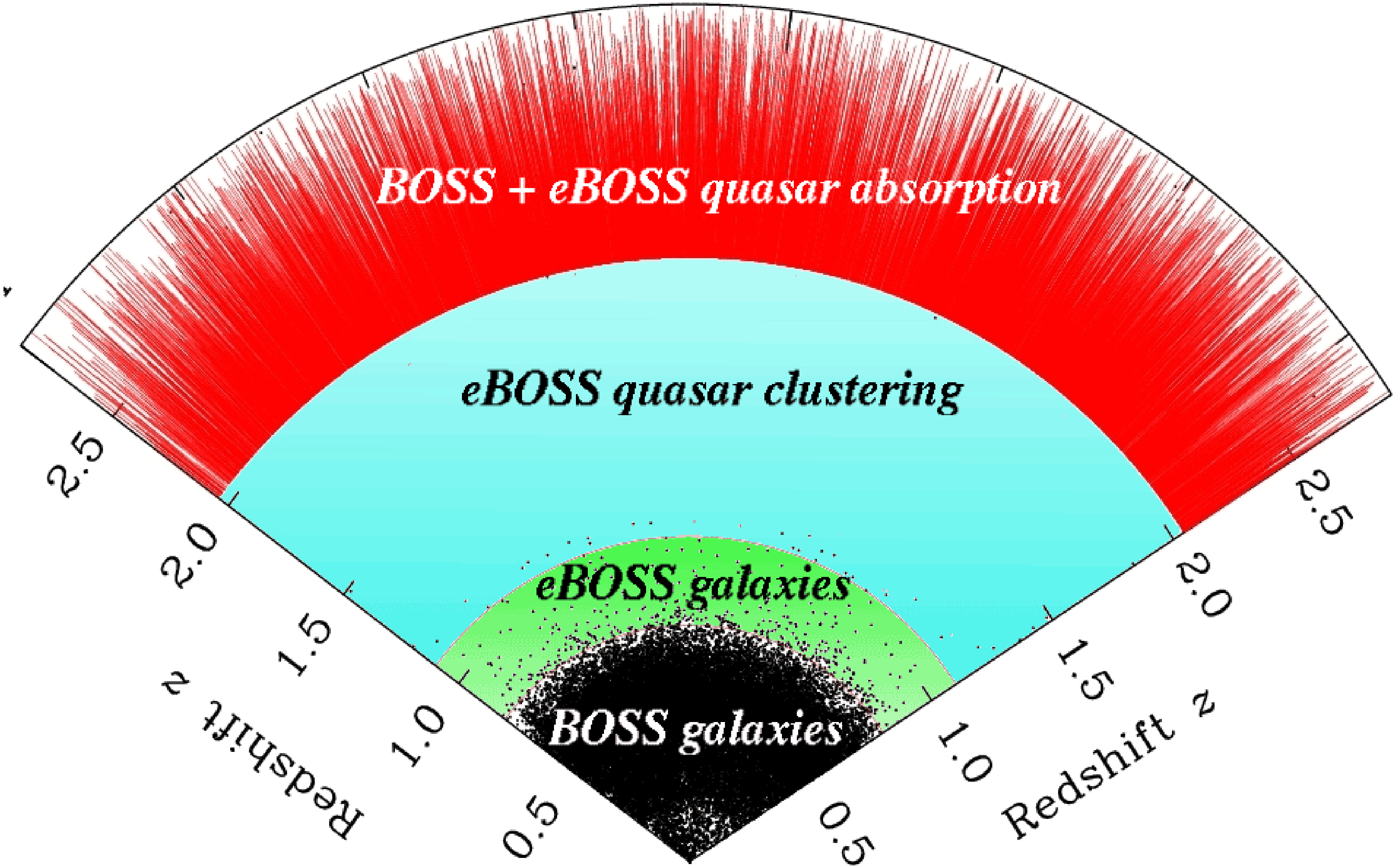
Distribución de galaxias en el experimento "Two Degree Field Galaxy Redshift Survey (2dF)". Estas observaciones miden la estructura en el universo correspondientes a distancias hasta 1000/h Mpc. 1 parsec = 3,0857 × 10^{16} m



Mapa del firmamento derivado del experimento SDSS-III. En el mapa se muestran los cúmulos de galaxias que son las estructuras más grandes en el universo. Crédito: SDSS-III Colaboración.



ALCANCE DEL EXPERIMENTO FUTURO eBOSS (2014-2020):



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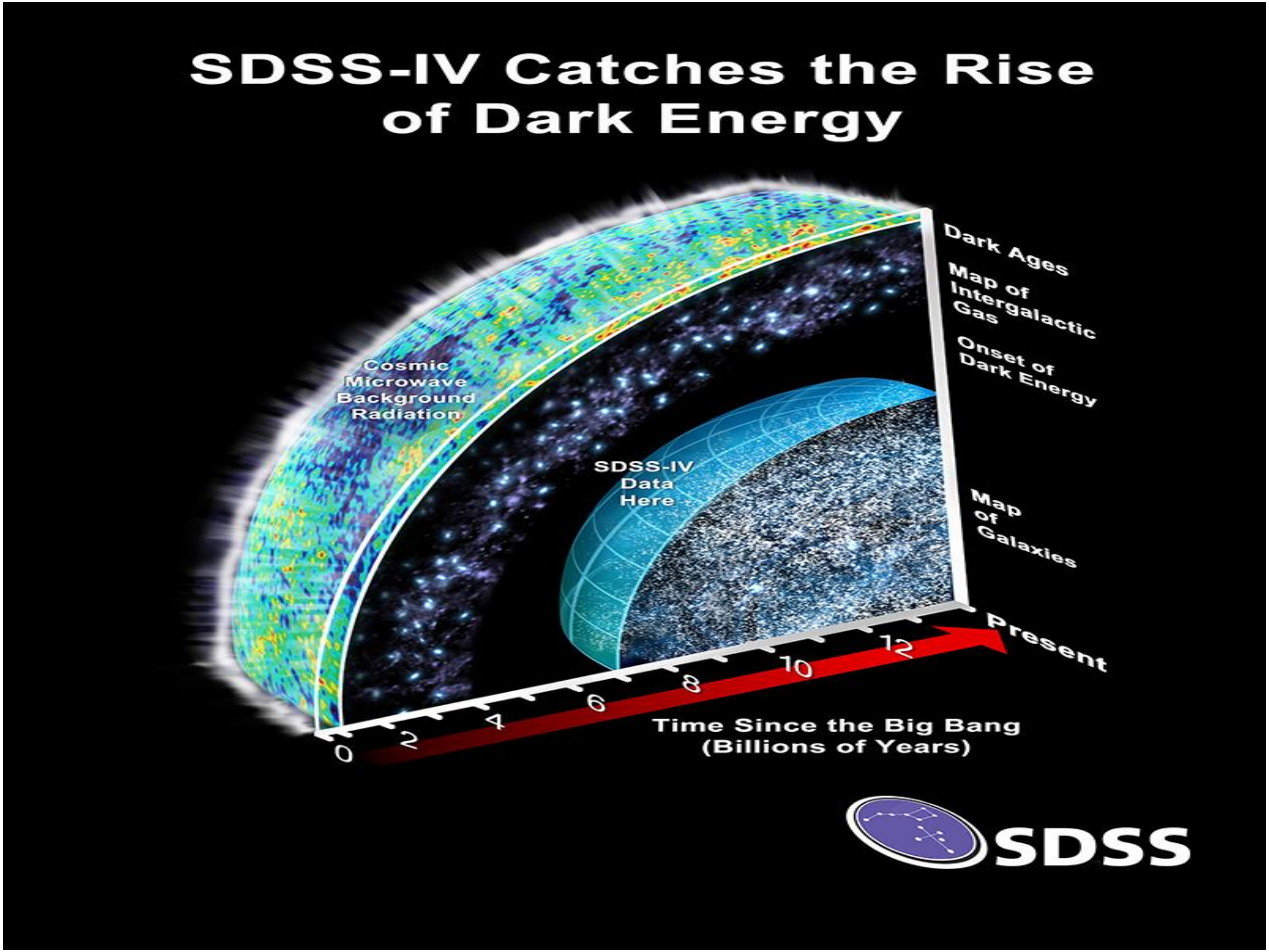


Un universo espacialmente isotrópico y homogéneo, y evolucionando en el tiempo significa que podemos representarlo como **R x Σ** donde Σ es una volumen espacial tridimensional isotrópico y homogéneo (maximalmente simétrico), y donde **R** representa la dirección en el tiempo:

$$ds^2 = -dt^2 + a^2(t)d\sigma^2$$

La variable **t** representa la coordenada de tiempo y la función **a(t)** representa el factor de escala.

Acorde con isotropía espacial y homogeneidad del universo, la métrica sobre Σ puede expresarse en coordenadas esféricas:

$$d\sigma^2 = \gamma_{ij}(u)du^i du^j = e^{2\beta(r)}dr^2 + r^2 d\Omega^2$$

La tres métrica maximalmente simétrica satisface:

$${}^{(3)}R_{ijkl} = k(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}) \quad \Longrightarrow \quad {}^{(3)}R = 6 \cdot k$$

$$\Longrightarrow \quad {}^{(3)}R_{jl} = 2k\gamma_{jl} \quad \Longrightarrow \quad \beta(r) = -\frac{1}{2}Ln(1 - kr^2)$$

Modelos cosmológicos Friedmann-Robertson-Walker (FRW) .

Cosmologias con volúmenes espaciales **Σ** homogéneos e isotrópicos.

La métrica FRW del Espacio-tiempo:

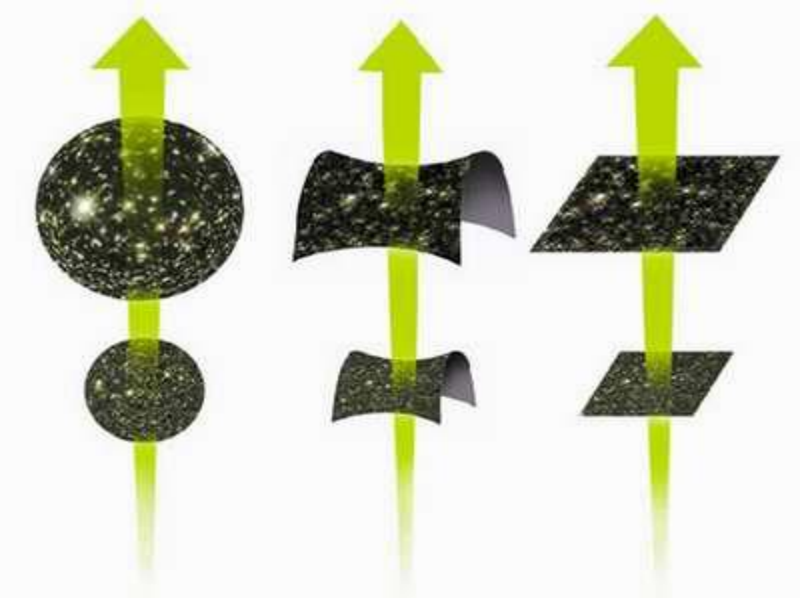
- Cerrado $k > 0$
- Abierto $k < 0$
- Plano $k = 0$

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sen^2\theta\, d\phi^2)\right]$$

Universo en expansión: **a(t) crece.**

Universo en contracción: **a(t) decrece.**

Un Universo inicialmente plano (abierto, cerrado) será siempre plano (abierto, cerrado) durante evolución temporal.



2.- El diagrama de Hubble: distancia modular vs corrimiento al rojo.

- Riess A. et al., Astron. J. 116, 1009 (1998), (50 Data).
- Perlmutter S. J. et al., Astrophys. J. 517, 565 (1999), (60 Data).
- Riess A. et al., Astrophys. J. 607, 665 (2004). [Gold D. (157) and Silver D.(29)].
- Astier P. et al., Astronomy & Astrophysics 447, 31 (2006). (SNLS) (71 Data).
- Kowalski M. et al., Astrophys. J. 686, 749 (2008). (SCP) (307 Data).
- Amanullah R. et al., Astrophys. J. 716, 712 (2010). (557 Data).

Ellos midieron la magnitud aparente de SNe Ia como una función del corrimiento al rojo.

$$\mu(z) \equiv m(z) - M$$

$\mu(z) \equiv$ Distancia Modular.

$m(z) \equiv$ Magnitude aparente.

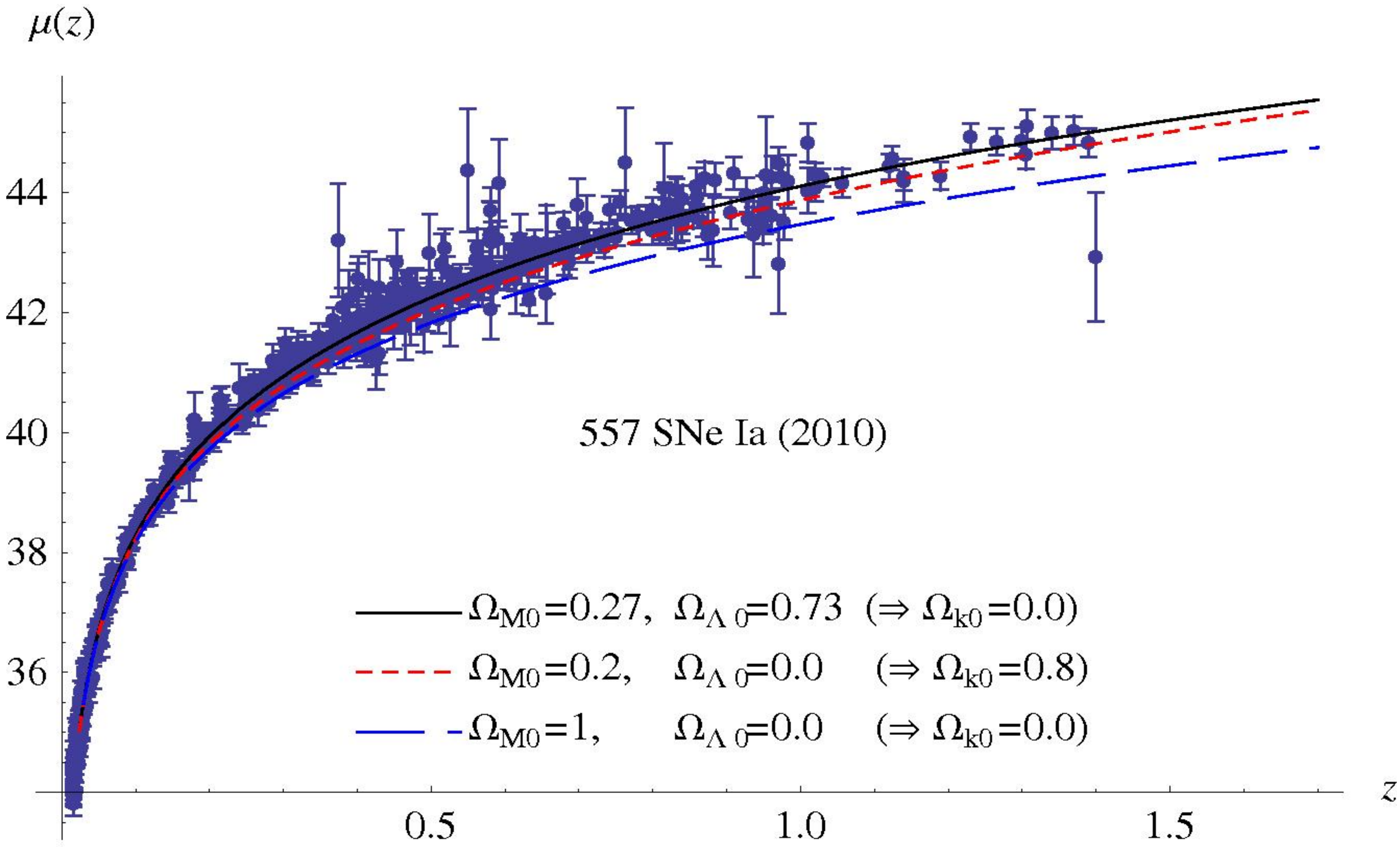
$M \equiv$ Magnitude absoluta.

$$z \equiv \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{e}}}{\lambda_{\mathrm{e}}} =$$

Corrimiento al rojo.

Diagrama Hubble (Muestra Union2, Amanullah et. al., 2010).

$$\mu(z) \equiv m(z) - M$$



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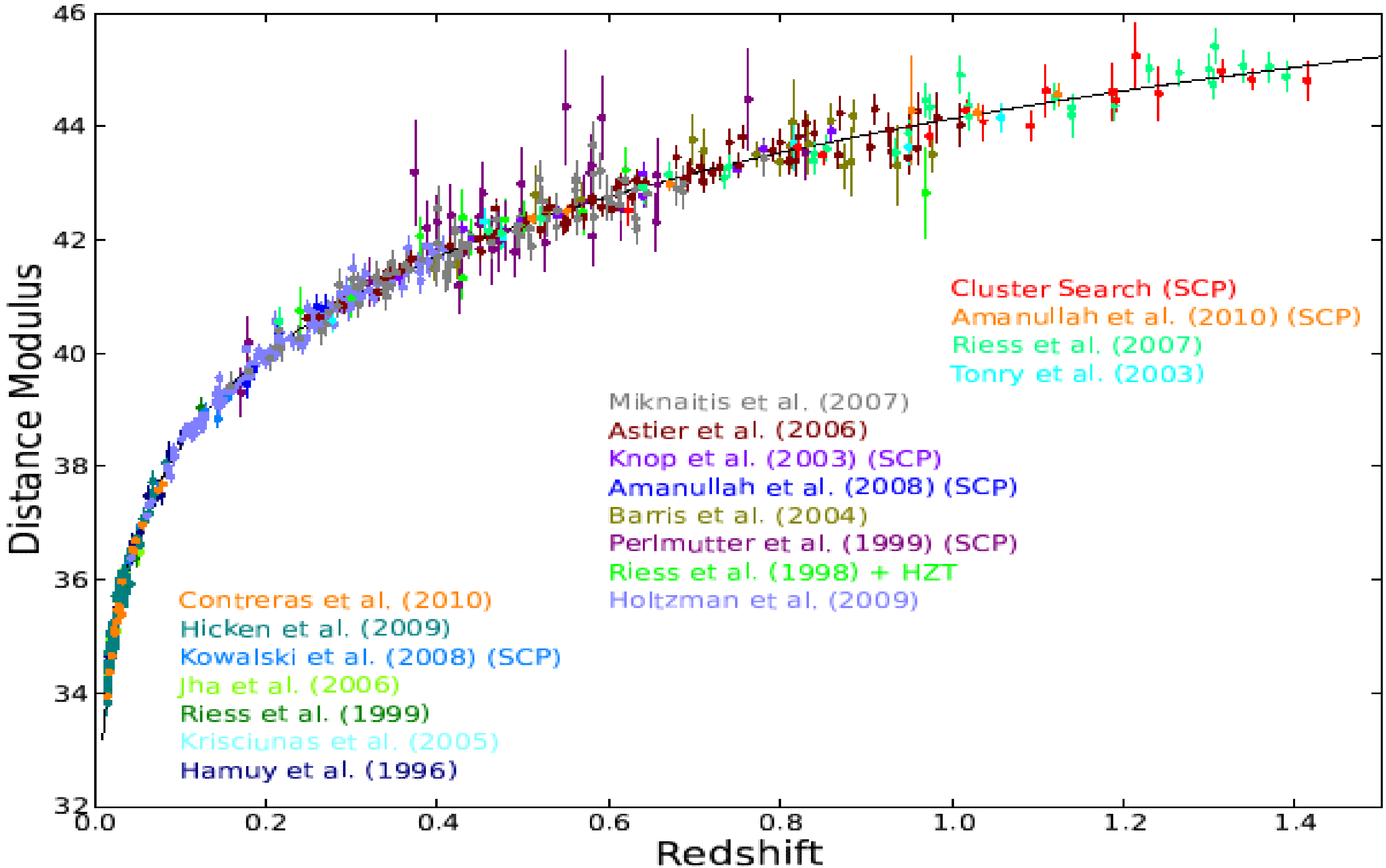
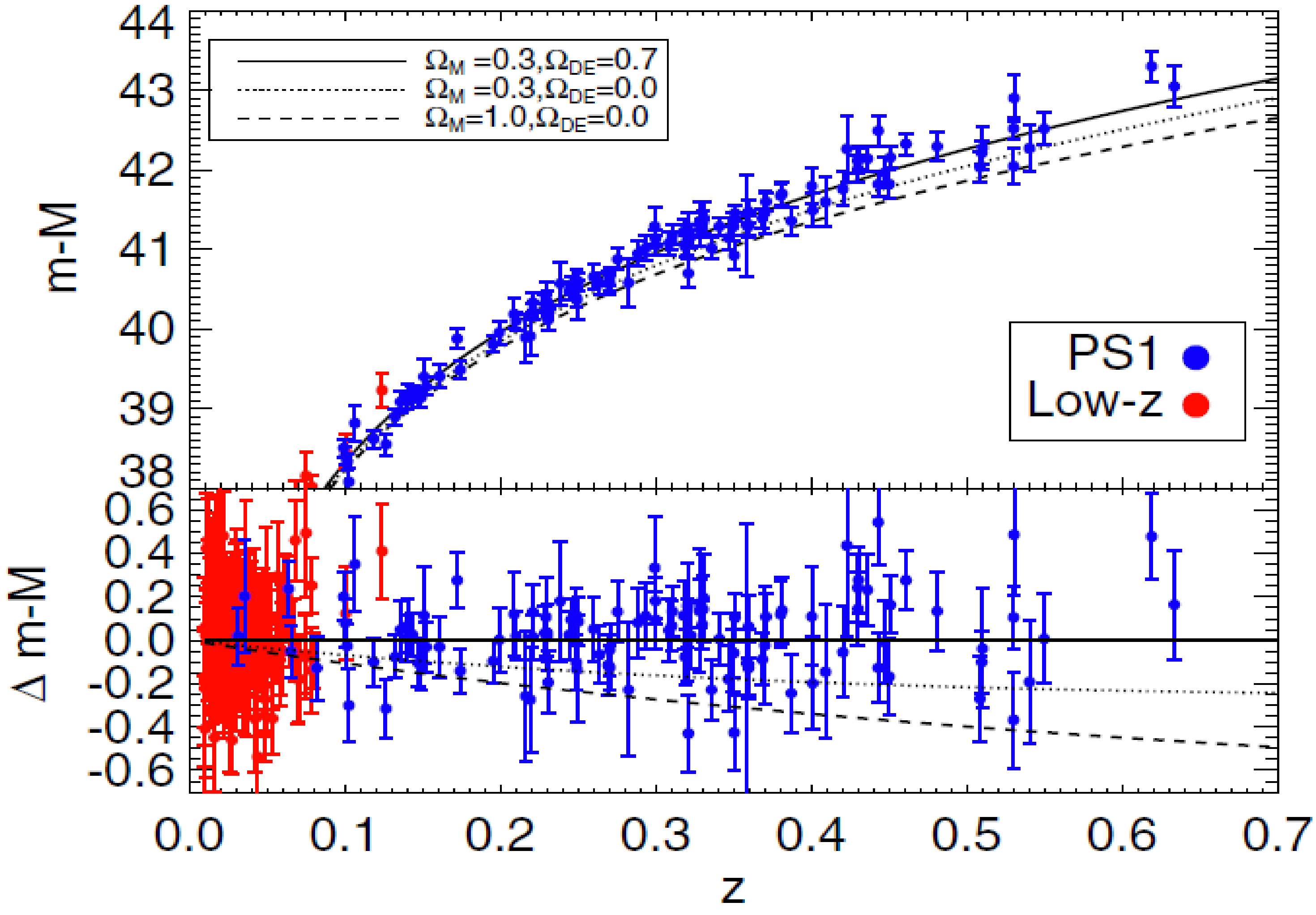


PAN-STARRS DATASET



3.- La distancia de luminosidad y la relación distancia modular vs corrimiento al rojo z.

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)},$$

El Parámetro de Hubble: razón de la expansión (velocidad del universo).

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2}$$

El Parámetro de desaceleración.

$$\frac{a(t)}{a_0} = a(t) \equiv \frac{1}{1+z} \quad \Longrightarrow \quad \frac{1}{H}\frac{dH}{dz} = \frac{[1+q(z)]}{(1+z)}$$

$$H_0 = H(t_0) = h \times 100 \ \frac{\text{km}}{\text{seg} \cdot \text{Mpc}}$$

La Constante de Hubble

$$h = 0.702 \pm 0.014 \qquad \text{WMAP} + \text{BAO} + H_0$$

$$h = 0.72 \pm 0.08 \qquad \text{HST (2001)}$$

Observaciones recientes

$$\mathrm{L} \equiv \frac{\text{energía radiada}}{\text{unidad de tiempo}}.$$

L = Luminosidad emitida por un objeto astrofísico tal como una Supernova Sne IA.

Luminosidad emitida:

$$L_e = \frac{N_e}{\delta t_e}\hbar\omega_e$$

Luminosidad Observada:

$$L_{obs} = \frac{N_{obs}}{\delta t_{obs}}\hbar\omega_{obs}$$

Pero:

$$N_{obs} \equiv N_e$$

$$1+z = \frac{\delta t_{obs}}{\delta t_e}$$

y

$$1+z \equiv \frac{\omega_e}{\omega_{obs}}$$

Entonces:

$$\frac{L_{obs}}{L_e} = \frac{1}{(1+z)^2}$$

Por otro lado, el flujo observado de la SNe se define como:

$$\boxed{f_{\mathrm{obs}} \equiv \frac{L_{\mathrm{obs}}}{4\pi\, d_{\mathrm{eff}}^{2}} = \frac{L_{\mathrm{e}}}{4\pi\, d_{\mathrm{eff}}^{2}\,(1+z)^{2}}}$$

$$\boxed{d_{\mathrm{eff}} \equiv a(z)\cdot r^{*} =}$$ Distance efectiva recorrida por fotones (luz).

Definimos una Distancia de Luminosidad: $$\boxed{d_{\mathrm{L}} \equiv (1+z)\, d_{\mathrm{eff}}}$$

Entonces tenemos: $$\boxed{f_{\mathrm{obs}} = \frac{L_{\mathrm{e}}}{4\pi\, d_{\mathrm{L}}^{2}}}$$

Cálculo de la distancia comóvil:

$$ds^2 = 0 = -c^2 dt^2 + a^2(t)\frac{dr^2}{1-kr^2}$$

La luz emitida viaja sobre geodésicas nulas.

Coordenadas comóviles de observador: $(r = 0, \theta = 0, \phi = 0)$

Coordenadas comóviles de emisor (SnI A) : $(r = r^*, \theta = 0, \phi = 0)$

$$\Longrightarrow \int_0^{r^*} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_e}^{t_o} \frac{dt}{a(t)}. = \int_0^z \frac{dz'}{H(z')}.$$

$$\int_0^{r^*} \frac{dr}{\sqrt{1-kr^2}} = \frac{1}{\sqrt{|k|}} \mathrm{sinn}^{-1}(\sqrt{|k|}r)\Big|_0^{r^*} = \frac{1}{\sqrt{|k|}} \mathrm{sinn}^{-1}(\sqrt{|k|}r^*),$$

$$\mathrm{sinn}(x) \equiv \begin{cases} \sin(x) & \text{si } k > 0 \\ x & \text{si } k = 0 \\ \sinh(x) & \text{si } k < 0 \end{cases}$$

$$\Longrightarrow \quad \frac{1}{\sqrt{|k|}}\mathrm{sinn}^{-1}(\sqrt{|k|}r^*) = \int_0^z \frac{dz'}{H(z')}.$$

$$\Longrightarrow \quad r^*(z) = \frac{1}{\sqrt{|k|}}\mathrm{sinn}\left[c\sqrt{|k|}\int_0^z \frac{dz'}{H(z')}\right]$$

Distancia comóvil entre emisor y observador

usando: $d_{\text{eff}} = a(t)r^*(z).$ y $d_L(z) \equiv d_{\text{eff}}(1+z).$

$$\Longrightarrow \quad \boxed{d_L(z) = \frac{(1+z)}{\sqrt{|k|}}\mathrm{sinn}\left[c\sqrt{|k|}\int_0^z \frac{dz'}{H(z')}\right].}$$

Distancia de Luminosidad.

Donde hemos definido:

$$\mathrm{sinn}(x) \equiv \begin{cases} \sin(x) & \text{si } k > 0 \\ x & \text{si } k = 0 \\ \sinh(x) & \text{si } k < 0 \end{cases}$$

La observación de Supernovas tipo IA:

Magnitud Absoluta:

$$M = -2.5\,\mathrm{Log}_{10}\left(\frac{L_e}{L_\Theta}\right) + 4.74$$

Magnitud Aparente:

$$m(z) = -2.5\,\mathrm{Log}_{10}\left(\frac{f_{obs}(z)}{f_{\Theta\, at\, 10pc}}\right) + 4.74$$

$$d_L^2 = \left(\frac{L_e}{4\pi f_{obs}}\right)$$

Relación entre distancia de Luminosidad vs luminosidad emitida y flujo observado.

$$\mu(z) \equiv m(z) - M = 5\log\left(\frac{d_L(z)}{1\,\mathrm{Mpc}}\right) + 25$$

Distancia Modular.

Donde la distancia de Luminosidad es:

$$d_L(z) = \frac{(1+z)}{\sqrt{|k|}}\mathrm{sinn}\left[c\sqrt{|k|}\int_0^z \frac{dz'}{H(z')}\right].$$

Tenemos la "Likelihood function" para *n datos* observados:

$$prob(D\,|\,X,I) \equiv \prod_{k=1}^{n} prob(\mu_k^{obs}\,|\,X,I) = \mathrm{A}\,Exp\left(-\frac{\chi^2}{2}\right)$$

Donde tenemos la distribución estadística Chi-cuadrada:

$$\chi^2(X) \equiv \sum_{k=1}^{n} \frac{[\mu_k^{teo}(z_k, X) - \mu_k^{obs}]^2}{\sigma_k^2}$$

Tenemos el Teorema de Bayes:

$$prob(X\,|\,D,I) \propto \mathrm{A}\,Exp\left(-\frac{\chi^2}{2}\right)\cdot prob(X\,|\,I)$$

Densidad de Probabilidad Posterior

"Likelihood density"

Densidad de Probabilidad previa.

5.- Ecuaciones cosmológicas de la Relatividad General.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\Lambda = 0$$

$$\Lambda \neq 0$$

Ecuaciones de Einstein.

Ecuaciones de Einstein con Constante Cosmológica.

La métrica FRW del Espacio-Tiempo:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sen^2\theta\, d\phi^2)\right]$$

Universo con varios Fluidos Perfectos:

$$T_{\mu\nu} = \rho_{total}\, u_\mu u_\nu + P_{total}(u_\mu u_\nu + g_{\mu\nu})$$

Con:

$$\rho_{total} = \sum_i \rho_i$$

$$P_{total} = \sum_i P_i$$

ρ_i = Densidad para el fluido i

P_i = Presión para el fluido i

u^μ = Velocidad de observador midiendo ρ_i y P_i

1ª Ecuación de Friedmann

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_{total} - \frac{k}{a^2}$$

2ª Ecuación de Friedmann

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{total} + 3P_{total}\right)$$

Ecuación para cada fluido:

$$\dot{\rho}_i + 3\left(\frac{\dot{a}}{a}\right)(\rho_i + P_i) = 0$$

Ecuación de Estado para cada fluido:

$$P_i = w_i \cdot \rho_i$$

Soluciones a la ecuación de cada fluido (con unidades con c=1)

Para: $w_i = \mathrm{Constant}$ $\Longrightarrow$

$$\rho_i = \rho_i^0 \cdot (1+z)^{3(1+w_i)} = \rho_i^0 \cdot \frac{1}{a^{3(1+w_i)}}$$

Definiendo:

$$\rho_{critic} = \frac{3H^2(t)}{8\pi G}$$

Densidad Crítica.

$$\Omega_i \equiv \frac{\rho_i}{\rho_{critic}}$$

Parámetros de densidad

$$\Omega_{total} = \sum_i \Omega_i$$

Parámetro de densidad total.

$$\Omega_k = -\left(\frac{k}{a^2(t)H^2(t)}\right)$$

Parámetro de densidad de curvatura.

1. Ecuación de Friedmann:

$$\Omega_{total} + \Omega_k = 1$$

Geometría del universo:

•Plano	k = 0	$\Omega_k = 0$	$\Omega_{total} = 1$
•Cerrado	k > 0	$\Omega_k < 0$	$\Omega_{total} > 1$
•Abierto	k < 0	$\Omega_k > 0$	$\Omega_{total} < 1$

Modelo del Universo compuesto por materia Barionica (polvo), materia oscura (polvo), Radiación de fotones de CMB, Constante Cosmológica, con ecuaciones de estado:

Fluido de materia bariónica y oscura: Polvo → $P_M \cong 0,\ w_M \cong 0$

Fluido de radiación de fotones: → $P_r = \frac{1}{3}\rho_r,\ w_r = \frac{1}{3}$

Fluido de Constante Cosmológica: → $P_\Lambda = -\rho_\Lambda,\ w_\Lambda = -1$

Donde la densidad de materia esta hecha de materia oscura y bariónica: → $\rho_M = \rho_{DM} + \rho_{BM}$

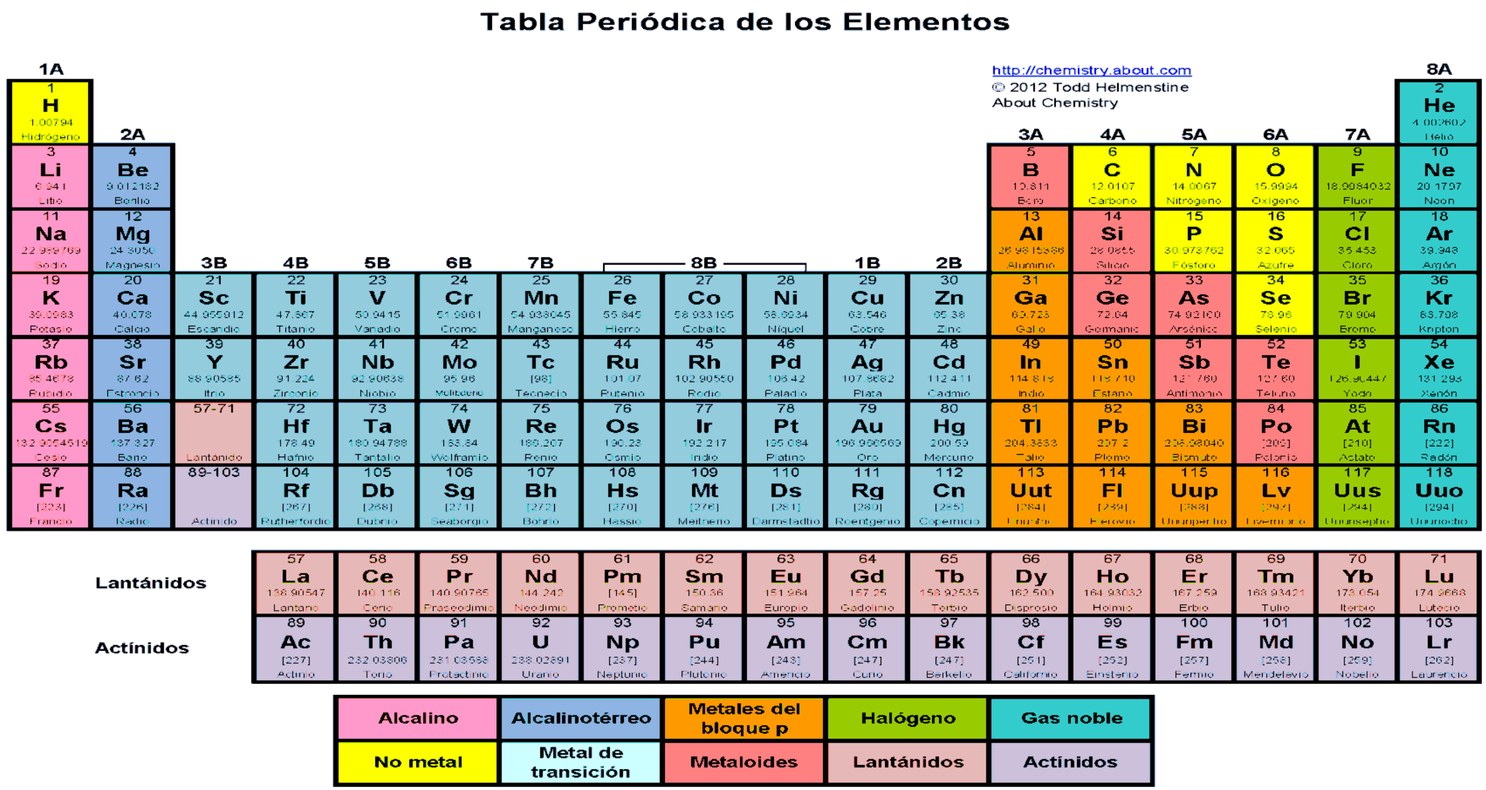
(1) ¿De que están hechas las estrellas, los planetas, el gas, el polvo de las galaxias?

(2) ¿Han existido siempre las galaxias, las estrellas, los planetas, el gas? NO !!!

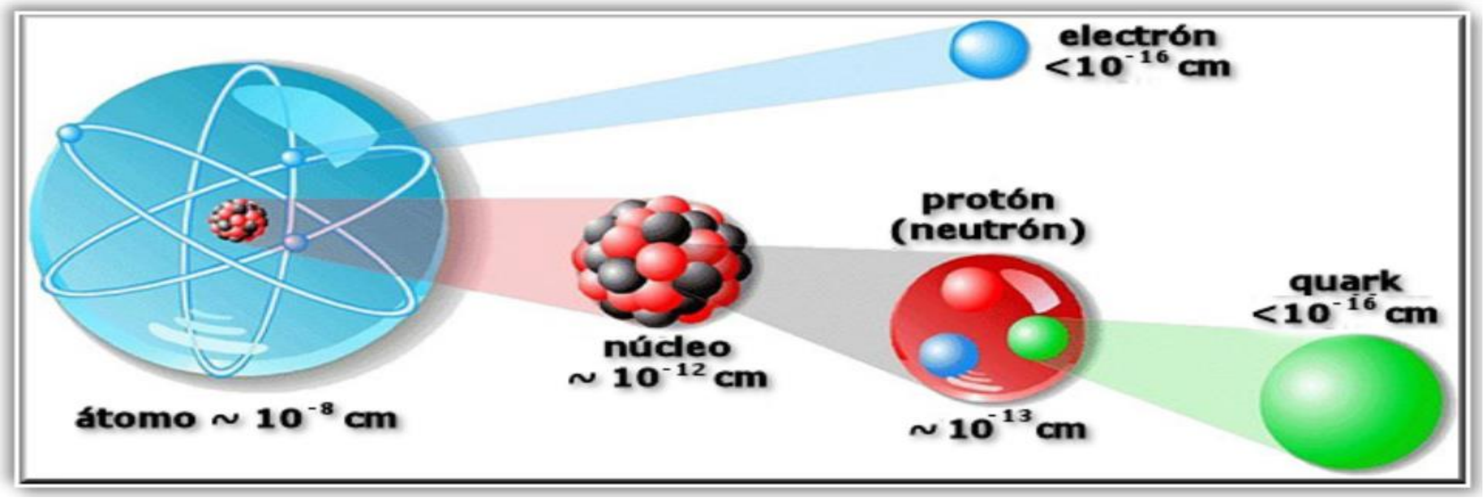
(3) ¿Qué fuerzas causaron su formación?

(1) Las estrellas, gas, planetas y el polvo están hechas de moléculas.

¿De que están hechas las moléculas?: Están hechas de átomos de la tabla periódica.



¿De que están hechos los átomos de la tabla periódica de los elementos?
Están hecho de electrones y núcleos que a su vez están formados de neutrones y protones.

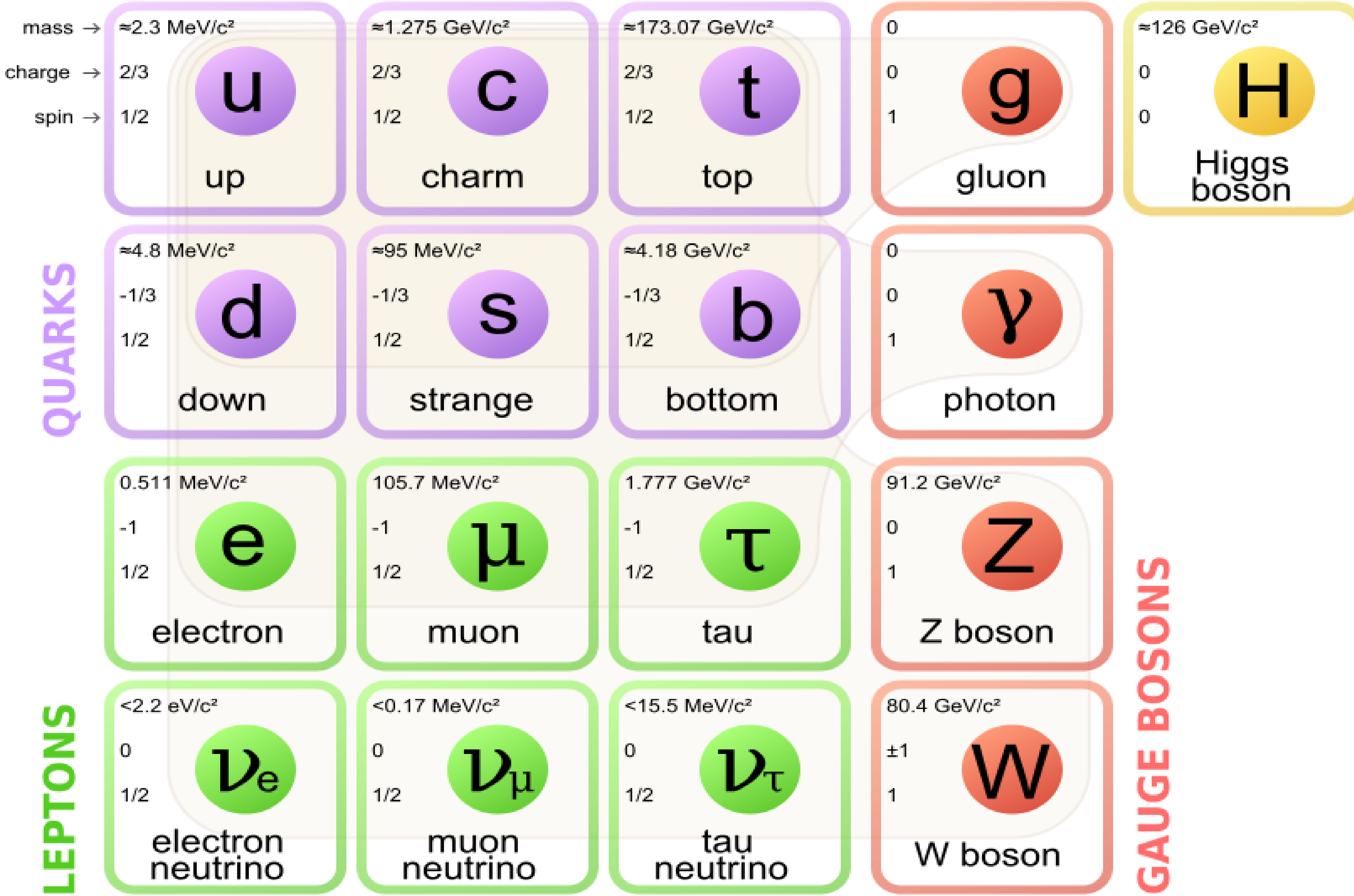


¿De qué están hechos los electrones?
No sabemos si los electrones están formados de otras partículas más pequeñas, hasta ahora los consideramos fundamentales (sin estructura).

¿De qué están hechos los protones y los neutrones?
Hoy en día sabemos que los protones y neutrones están formados cada uno de 3 partículas más pequeñas llamadas Quarks.
Hemos descubierto la existencia de 6 tipos distintos de Quarks.

Además de estás partículas, ¿existen más partículas elementales?

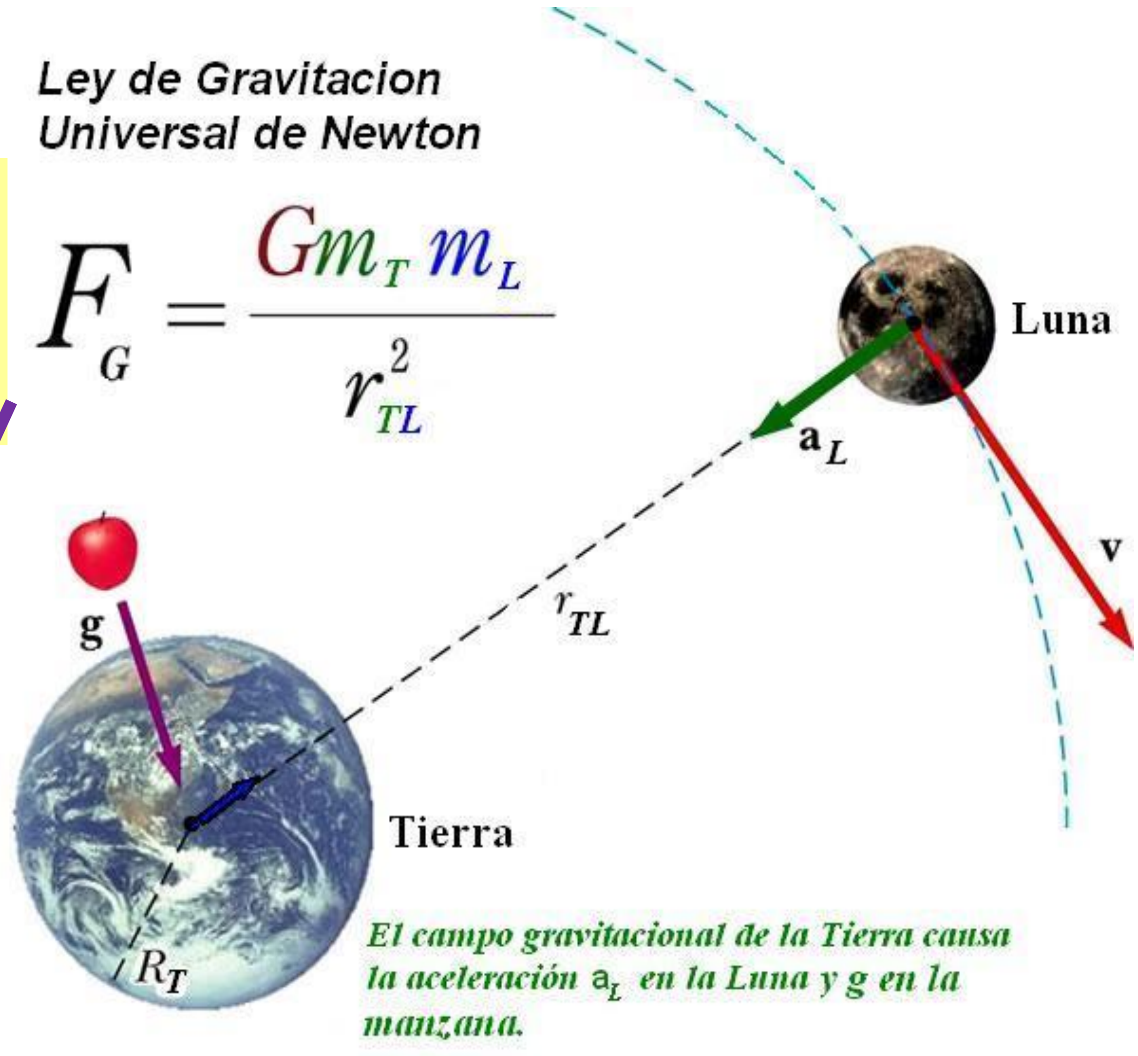
Estrellas, planetas, gas, moléculas y átomos de la tabla periódica están hechos de partículas del MODELO ESTÁNDAR DE PARTÍCULAS ELEMENTALES.



La ley de fuerza gravitacional Newtoniana:

Aceleración centrípeta

$$m_L \cdot \frac{v^2}{r_{TL}} = m_L \cdot a_L =$$

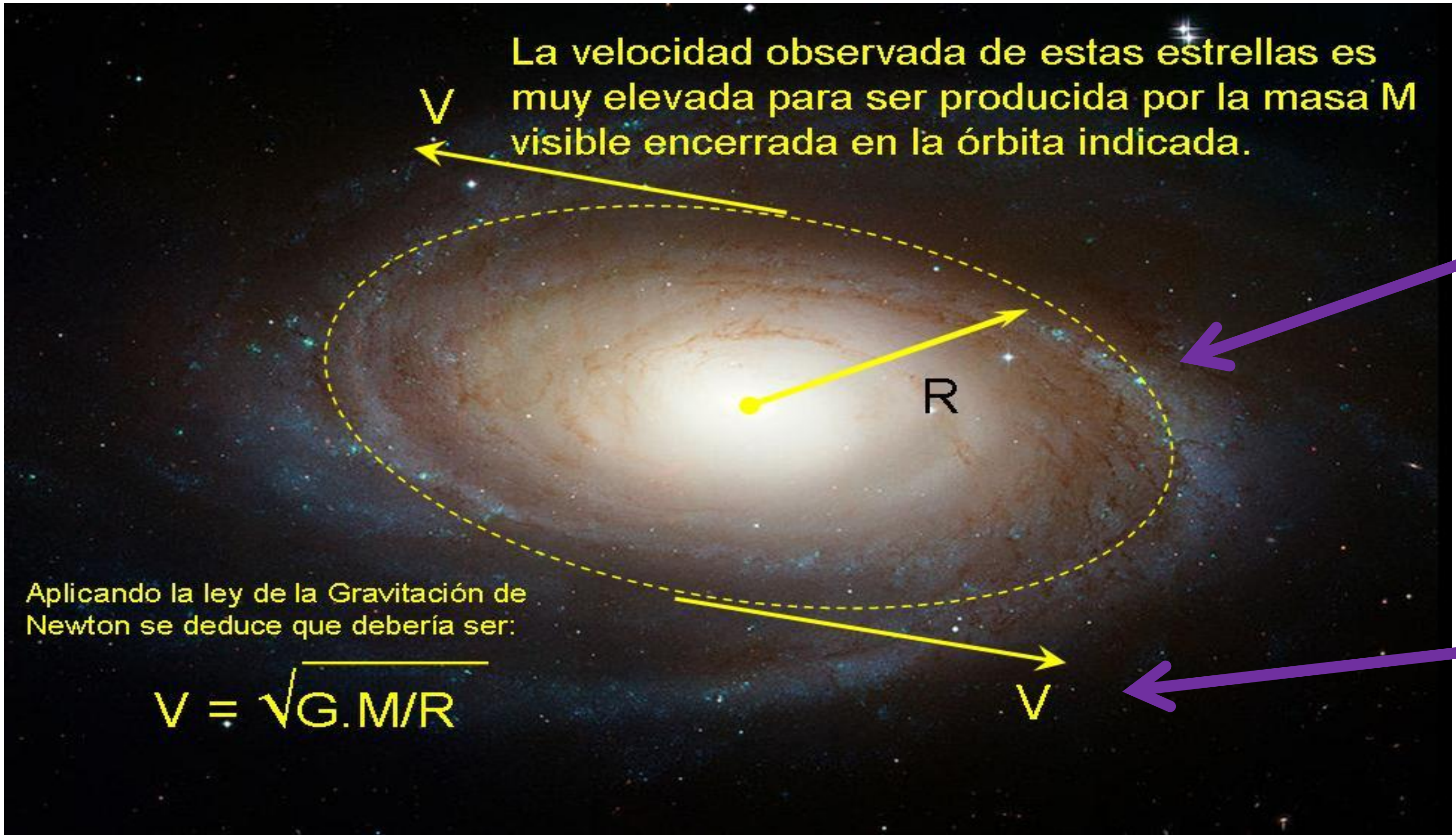


Velocidad circular

$$v^2 = \frac{G m_T}{r_{TL}}$$

Las estrellas, gas y planetas en una galaxia se mantienen unidas por la acción de la fuerza gravitacional Newtoniana.

Las componentes de MASA VISIBLES de una galaxia espiral son:
(1) Estrellas luminosas del disco galáctico. (Emite fotones en el rango visible)
(2) Gas de hidrógeno neutral (HI). (Emite fotones de radio de 21 cm de longitud)



Masa dentro del círculo.

Velocidad circular

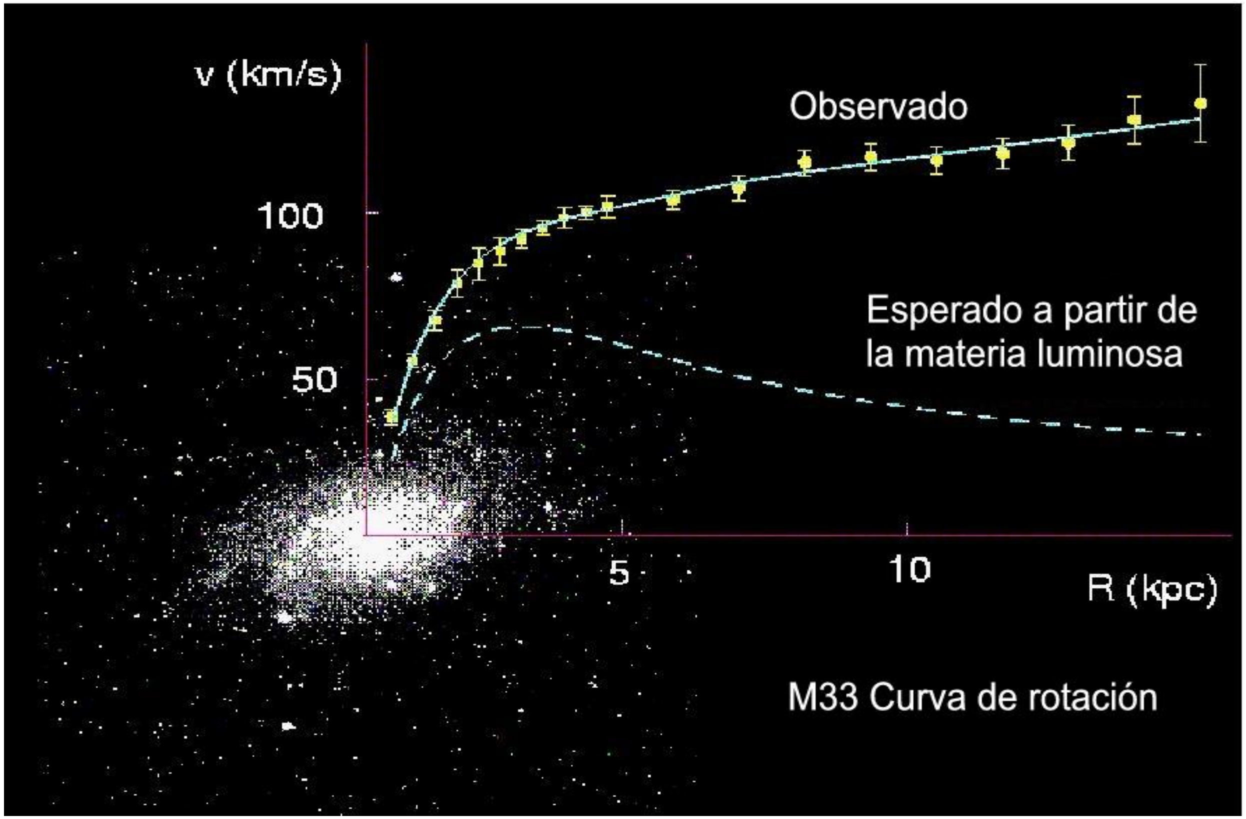
CURVAS DE ROTACIÓN GALÁCTICAS EN GALAXIAS ESPIRALES

Curva de datos de velocidad circular (V) de partículas (estrellas o gas) rotantes contra el radio (R) de su órbita circular.

$$m = \text{masa total encerrada en el radio R.}$$

$$m_L = \text{masa de estrellas y de gas.}$$

Velocidad observada



$$\mathrm{v} = \sqrt{\frac{Gm}{\mathrm{R}}}$$

$$\mathrm{v}_L = \sqrt{\frac{Gm_L}{\mathrm{R}}}$$

$$\mathrm{V}_L \neq \mathrm{V}$$

¿Porqué no coinciden?

$$V_L \neq V$$

IDEA: NO COINCIDEN PORQUE FALTA MASA !!

LA LLAMAREMOS MATERIA OBSCURA:

MATERIA DETECTADA SOLAMENTE POR MEDIO DE SU FUERZA GRAVITACIONAL PERO QUE NO EMITE FOTONES (NO EMITE ONDAS ELECTROMAGNÉTICAS).

Introducimos un halo hipotético hecho de materia obscura.

$$m_h = \text{masa de halo de materia obscura.}$$

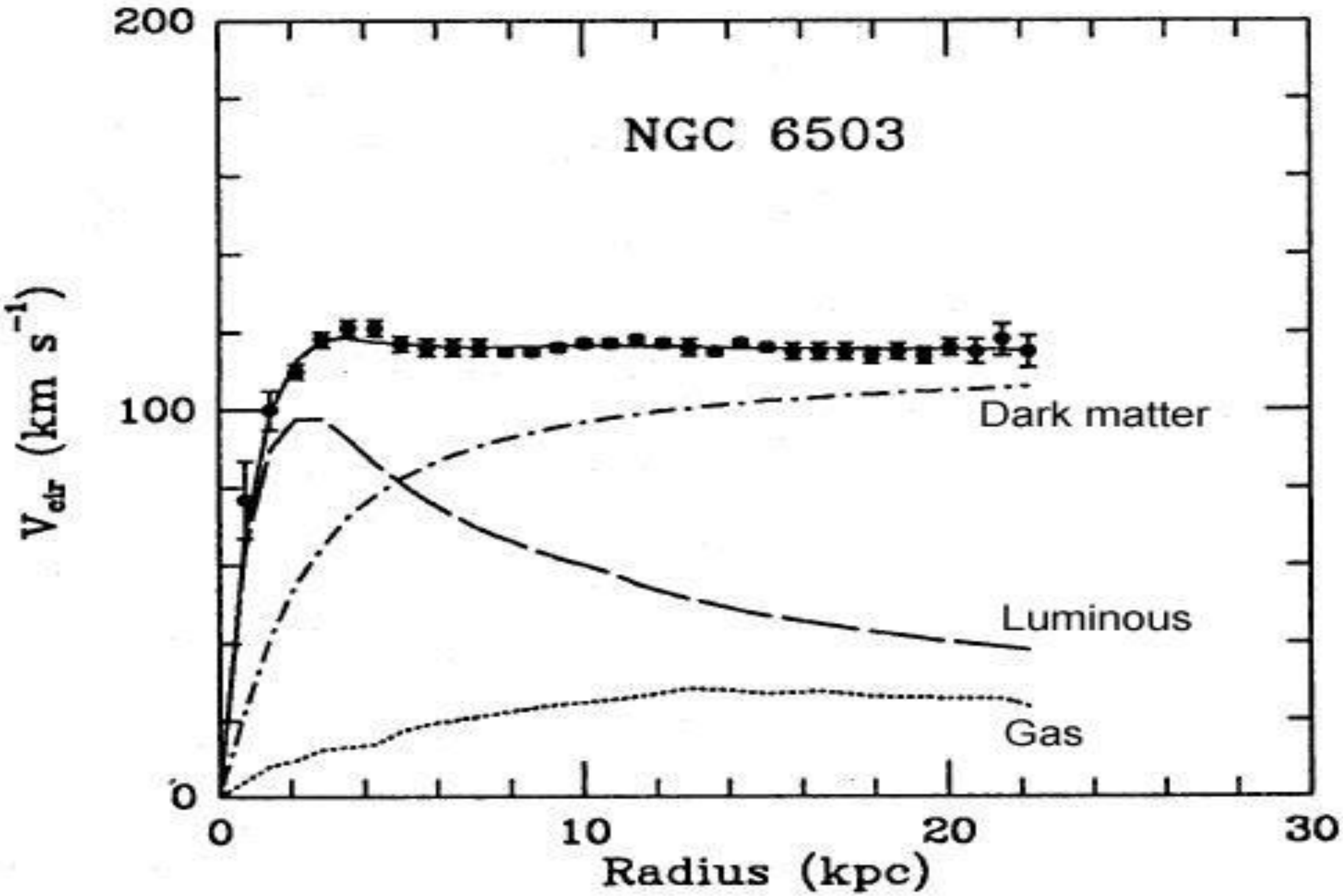
Velocidad debida al halo de materia obscura. →

$$\mathrm{v_h} = \sqrt{\frac{Gm_h}{\mathrm{R}}}$$

Velocidad observada al cuadrado. →

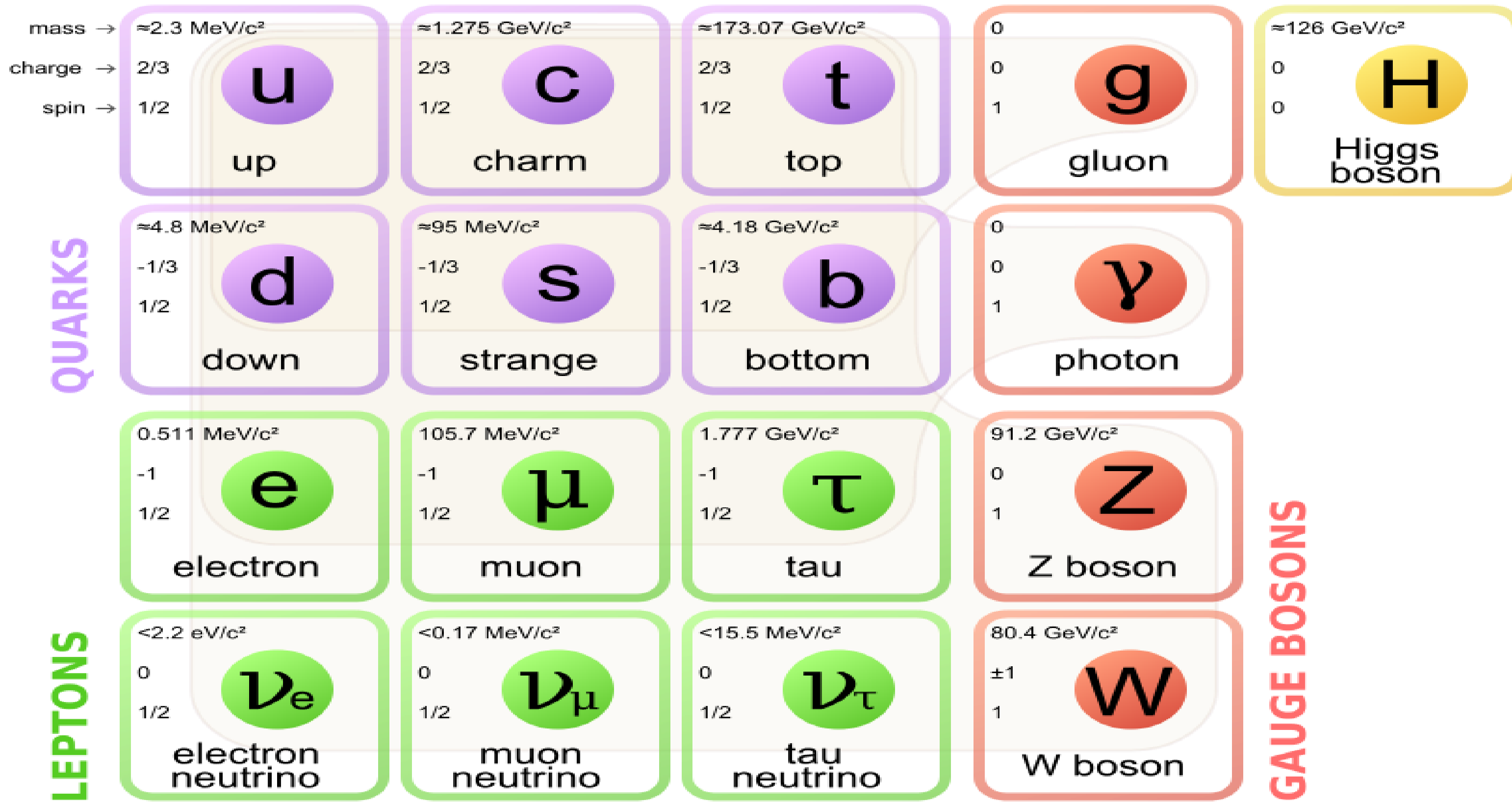
$$\mathrm{v}^2 = \mathrm{v}_L^2 + \mathrm{v}_\mathrm{h}^2 = \frac{G}{\mathrm{R}}\left(m_L + m_h\right) = \frac{Gm}{\mathrm{R}}$$

CURVA DE ROTACIÓN TÍPICA DE GALAXIAS ESPIRALES.



¿De qué está hecha la materia obscura? **!! NO SABEMOS !!**

!! Lo que si sabemos es que la materia obscura no puede estar hecha de partículas del modelo estándar de partículas elementales !!



Soluciones para la densidad de cada fluido:

Fluido de materia barionica y obscura: → $$\rho_M(z) = \rho_M^0 \cdot (1+z)^3$$

Fluido de radiación de fotones: → $$\rho_r(z) = \rho_r^0 \cdot (1+z)^4$$

Fluido de Constante Cosmológica: → $$\boxed{\rho_\Lambda(z) = \rho_\Lambda^0}$$

Donde la densidad de materia está compuesta de materia barionica y obscura: → $$\rho_M = \rho_{DM} + \rho_{BM}$$

Densidad de constante cosmológica : → $$\boxed{\rho_\Lambda^0 = \frac{\Lambda}{8\pi G}}$$

Ecuaciones cosmológicas:

1ª Ecuación Friedmann:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\left[\rho_r + \rho_M + \rho_\Lambda^0\right] - \frac{k}{a^2}$$

2ª Ecuación Friedmann:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\cdot\left[2\rho_r + \rho_M - 2\rho_\Lambda^0\right]$$

Pero sabemos que: $\rho_M > 0\,,\ \rho_r > 0$

Y asumimos: $\rho_\Lambda^0 > 0$

Expansión desacelerada:

$$2\rho_r + \rho_M - 2\rho_\Lambda^0 > 0$$

Expansión acelerada:

$$2\rho_r + \rho_M - 2\rho_\Lambda^0 < 0$$

Evidencia para la existencia de una constante cosmológica.

Para nuestro universo, usamos un modelo compuesto por materia barionica y obscura, radiación de fotones y constante cosmológica, entonces la 1.- ecuación de Friedmann es:

$$\Omega_M + \Omega_\Lambda + \Omega_r + \Omega_k = 1$$

Y el correspondiente Parámetro de Hubble:

$$H(z) = H_0 \sqrt{\Omega_M^0 (1+z)^3 + \Omega_\Lambda^0 + \Omega_r^0 (1+z)^4 + \Omega_k^0 (1+z)^2}$$

$$\Omega_r^0 = \frac{8\pi G}{3H_0^2} \rho_r^0 = 2.5 \times 10^{-5} h^{-2}$$

Para el análisis de datos de SNe la despreciamos la contribución de radiación de fotones porque es subdominante en el rango de tiempo considerado. **Consideramos un universo formado por Materia Barionica, Materia Obscura, y Constante Cosmológica.**

Evidencia para la existencia de una Constante Cosmologica.

Universo formado por Materia, Constante Cosmológica y con Curvatura:

$$d_L(z) = \frac{c(1+z)}{H_0 \left|1-\Omega_M^0-\Omega_\Lambda^0\right|^{1/2}} \, senn\left(\left|1-\Omega_M^0-\Omega_\Lambda^0\right|^{1/2} \int_0^z \frac{du}{\tilde{H}(u)} \right)$$

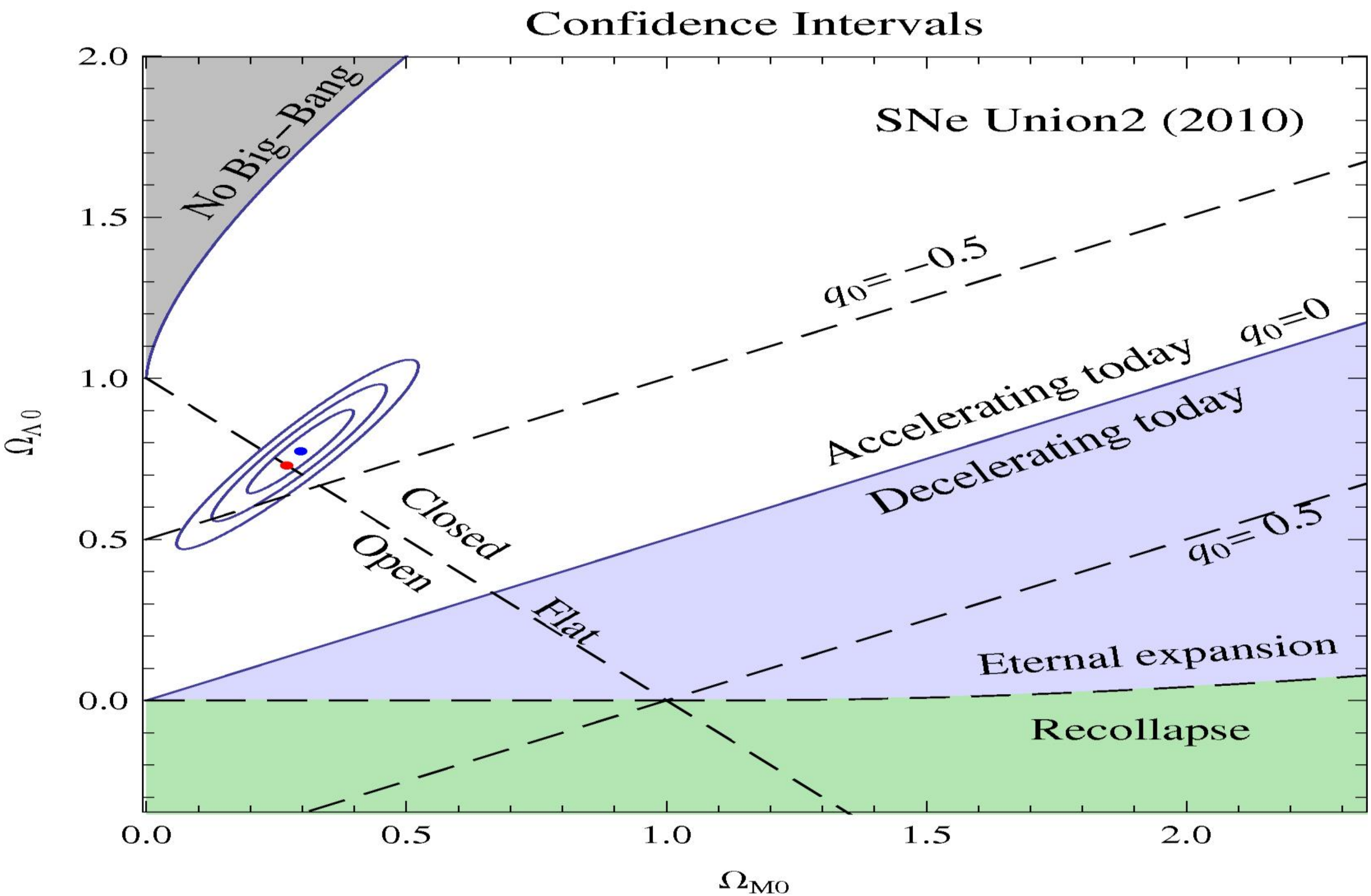
Donde:

$$\tilde{H}(z) = \sqrt{\Omega_M^0(1+z)^3+\Omega_\Lambda^0+(1-\Omega_M^0-\Omega_\Lambda^0)(1+z)^2}$$

Densidad de probabilidad posterior marginalizada sobre la constante de Hubble para un universo formado por materia (oscura y bariónica), constante cosmológica y curvatura:

$$P(\Omega_M^0,\Omega_\Lambda^0) \equiv B\cdot\exp\left[-\frac{\chi^2(\Omega_M^0,\Omega_\Lambda^0)-\chi^2_{\min}}{2}\right] = A\cdot\int_0^\infty \exp\left[-\frac{\tilde{\chi}^2(H_0,\Omega_M^0,\Omega_\Lambda^0)}{2}\right]dH_0$$

Restricciones sobre densidad de constante Cosmológica y de materia.



The total χ^2-function

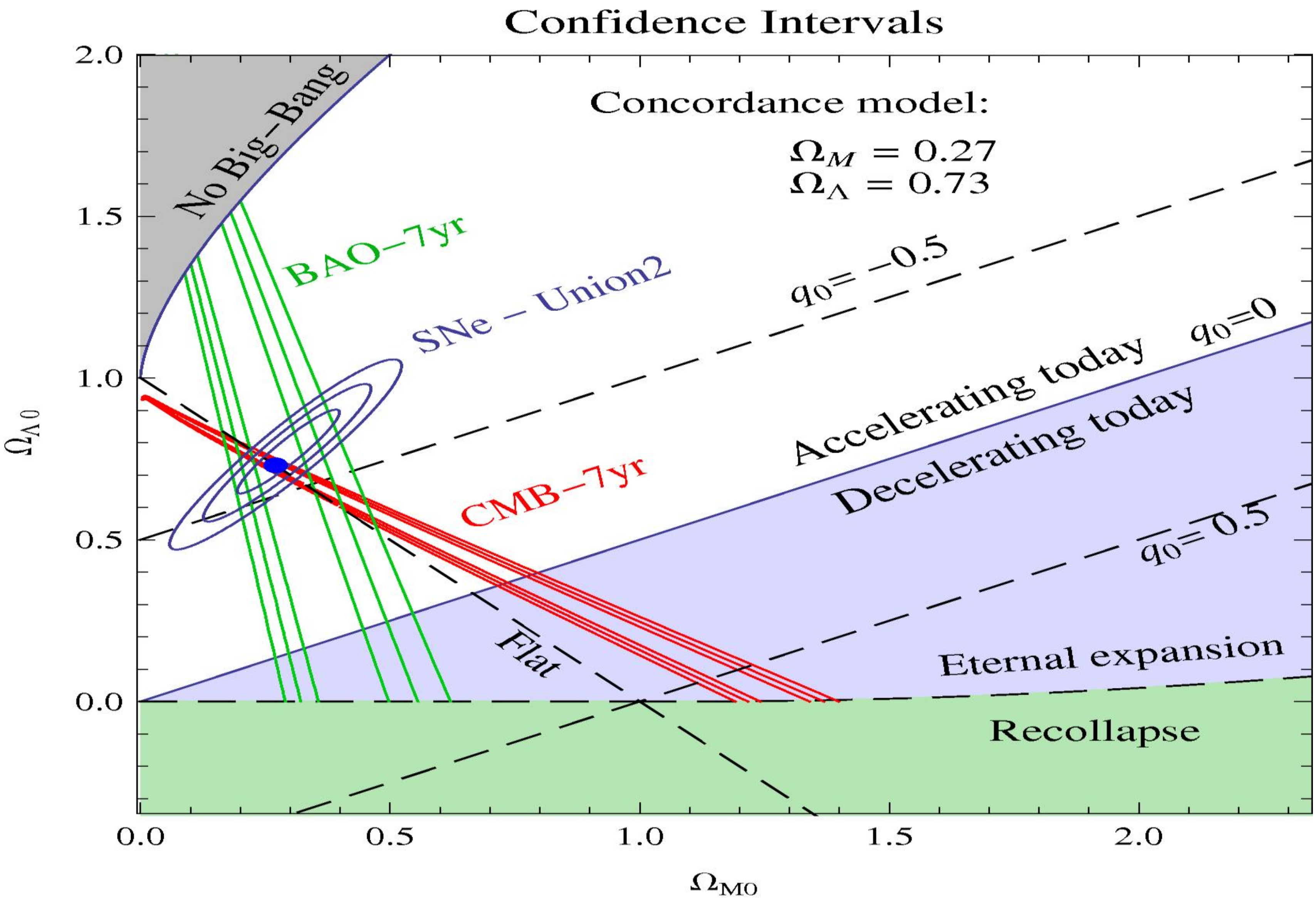
$$\chi^2_{\mathrm{SNe}}(z,\mathbf{X}) \equiv \sum_{k=1}^{n} \frac{[\mu_k^{\mathrm{theory}}(z,\mathbf{X}) - \mu_k^{\mathrm{observ}}]^2}{\sigma_k^2}$$

$$\chi^2_{\mathrm{CMB}}$$

$$\chi^2_{\mathrm{BAO}}$$

$$\boxed{\chi^2 = \chi^2_{\mathrm{SNe}} + \chi^2_{\mathrm{CMB}} + \chi^2_{\mathrm{BAO}}}$$

Cotas sobre los parámetros de densidad de Constante Cosmológica y densidad de materia.



Nucleosíntesis del Modelo de Big Bang.

Parámetro inferido de densidad de materia bariónica presente: → $\Omega^0_{bar} = 0.04 \pm 0.02$

Parámetro inferido de densidad de materia presente: → $\Omega^0_{\mathrm{M}} = 0.3 \pm 0.1$

Existencia de materia obscura:

$$\Omega^0_{obscura} = \Omega^0_{\mathrm{M}} - \Omega^0_{bar} \approx 0.26 \pm 0.1$$

Composición del universo: Modelo de Concordancia.

Porcentaje de la densidad crítica presente:

- Materia Barionica (átomos): $\approx$ 5 %
- Materia obscura: $\approx$ 25 %
- Radiación de fotones: $\approx$ 0.005 %
- Constante Cosmológica: $\approx$ 70 %
- Otras componentes (neutrinos, electrones) $\approx$ 0 %

Densidad Crítica presente:

$$\rho^{0}_{critica} = 1.88 \times 10^{-29} h^{2} \frac{gr}{cm^{3}}$$

Del vacío cuántico del Modelo Estándar de partículas elementales:

$$\rho_\Lambda^{EW} \approx 3\times10^{47}\, erg/cm^3$$

$$\rho_\Lambda^{QCD} \approx 1.6\times10^{36}\, erg/cm^3$$

Densidad de Planck

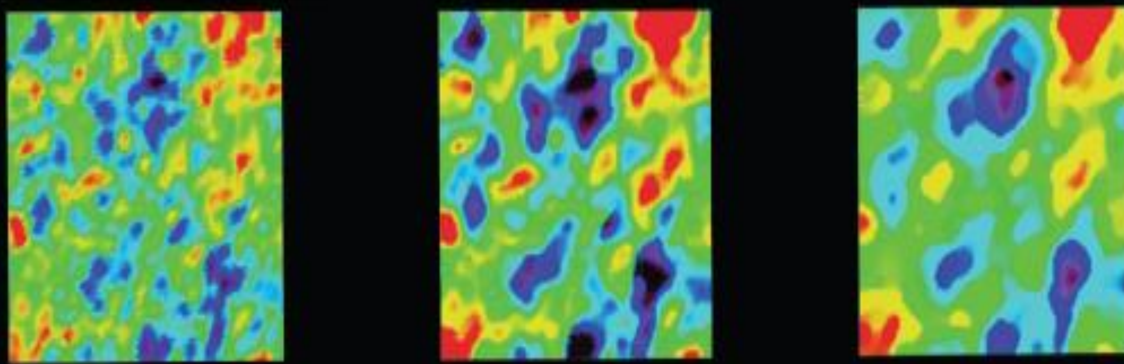
$$\rho_{\Lambda(\text{planck})}^{total} \approx 2\times10^{110}\, erg/cm^3$$

Densidad observada para la constante cosmológica:

$$\Omega_\Lambda^0 = \frac{8\pi G}{3H_0^2}\rho_\Lambda^0 \approx 0.7 \longrightarrow \rho_\Lambda^{observada} \approx 2\times10^{-10}\, erg/cm^3$$

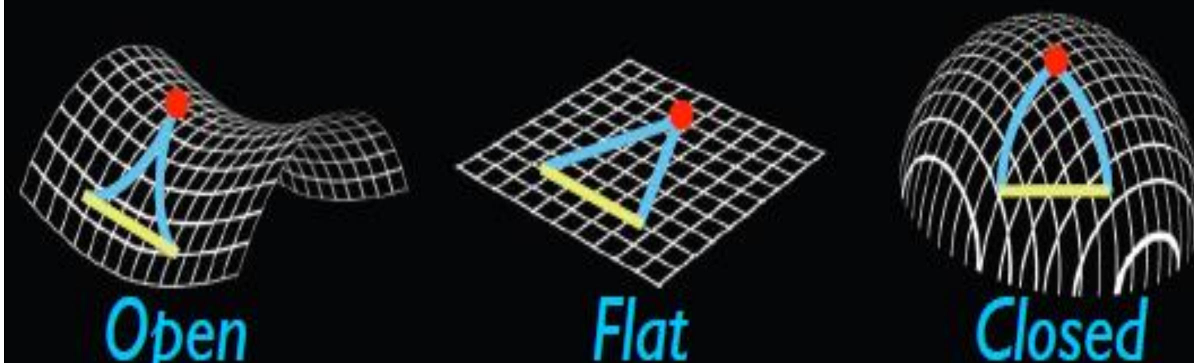
Una razón de 120 órdenes de magnitud !!!

Problema de la Constante Cosmológica !!!



$\Omega < 1 \Rightarrow \theta_c < 1^\circ$ $\Omega = 1 \Rightarrow \theta_c \simeq 1^\circ$ $\Omega > 1 \Rightarrow \theta_c > 1^\circ$

The physical size of the fluctuations is the horizon size at the last scattering surface.

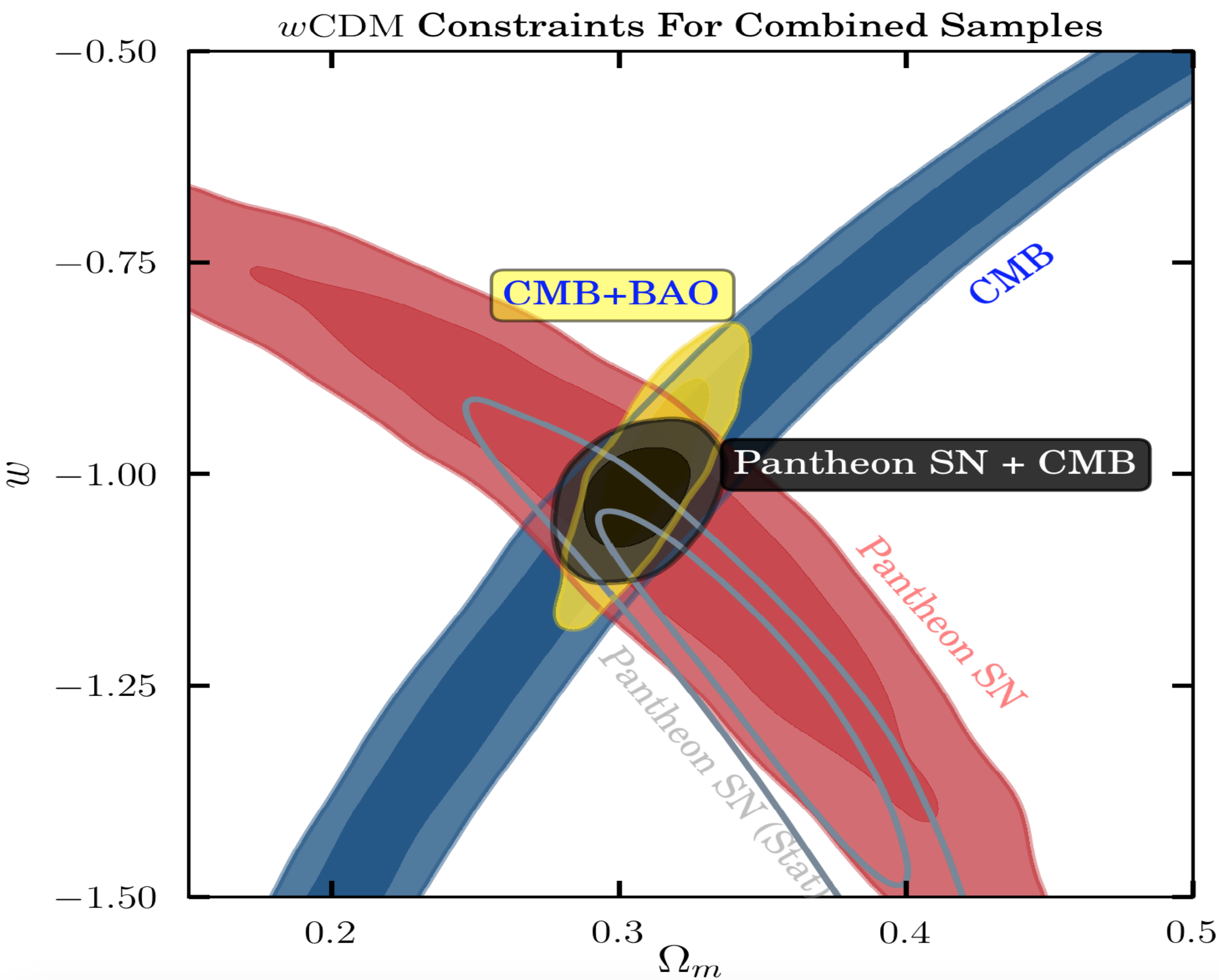


The geometry of the Universe determines the angular size of the fluctuations.

$$\Omega \equiv \frac{\text{Energy in the Universe}}{\text{Energy required for flatness}} = 1.005 \pm 0.007 \text{ today}$$

Adrienne Erickcek

PARÁMETRO W CONSTANTE DE ECUACIÓN DE ESTADO DE ENERGÍA OBSCURA



Clasificación de Modelos de Energía Obscura.

WEC implica: → $\rho \geq 0 \quad \text{and} \quad \rho + P \geq 0$ **WEC**

DEC implica: → $\rho \geq 0 \quad \text{and} \quad |P| \leq \rho$ **DEC**

Para Energía Obscura, la DEC implica:

$\rho_{DE} > 0$ **AND** $-\rho_{DE} \leq P_{DE} \leq \rho_{DE} \Rightarrow -1 \leq w_{DE} \leq 1$

Combinando con la condición mínima para aceleración positiva: $\Rightarrow$ $-1 \leq w_{DE} < -\frac{1}{3}$

$-1 \leq w_{DE} < -\frac{1}{3}$ ← **QUINTESSENCE DARK ENERGY** → **DEC**

$w_{DE} < -1$ ← **PHANTOM DARK ENERGY** → **NON-DEC**

Both behaviors ← **QUINTOM DARK ENERGY**

Reasonable Energy Conditions on Classical Matter-Energy.

Weak Energy Condition: For Classical Matter the Energy Density is Nonnegative

$$(1)\quad \rho = T_{\mu\nu} t^{\mu} t^{\nu} \geq 0 \;,\; \forall\, t^{\mu} = \text{Futured Directed Timelike vector}$$

WEC

Dominant Energy Condition believed to hold for physically reasonable energy:

$$(2)\quad -T^{\mu}_{\nu} t^{\nu} = \text{Future directed non-spacelike vector,}$$
$$\forall\, t^{\nu} = \text{Future directed timelike vector}$$

DEC

DEC $\Longrightarrow$ **WEC**

$$-T^{\mu}_{\nu} t^{\nu} = \text{Physically represents the energy-momentum current density}$$
$$\text{of matter-energy observed by the timelike vector.}$$

$\Longrightarrow$

Interpretation: DEC means that the speed of energy flow of matter-energy is always less than the speed of light.

Dividing Curve between perpetual expansion and eventual recollapse.

The Friedmann equation is written as:

$$\frac{H^2}{H_0^2} = \frac{\Omega_M^0}{a^3} + \Omega_\Lambda^0 + \frac{\Omega_k^0}{a^2}$$

$$\Omega_k^0 = 1 - \Omega_M^0 - \Omega_\Lambda^0$$

We can rewrite the Friedmann equation as:

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We obtain a cubic equation for the scale factor at turnaround: a_*

$$\Omega_\Lambda^0 \cdot a_*^3 + (1-\Omega_M^0 - \Omega_\Lambda^0)\cdot a_* + \Omega_M^0 = 0$$

Solving this cubic equation we find that the value of Ω_Λ^0 for which the universe will expand forever is given by:

$$\begin{array}{lll} \Omega_\Lambda^0 \geq 0 & \text{if} & 0 \leq \Omega_M^0 \leq 1 \\ \Omega_\Lambda^0 \geq 4\cdot\Omega_M^0 \cdot \cos^3\left[\frac{1}{3}\cdot\cos^{-1}\left(\frac{1-\Omega_M^0}{\Omega_M^0}\right)+\frac{4\cdot\pi}{3}\right] & \text{if} & \Omega_M^0 > 1 \end{array}$$

Dividing curve between perpetual expansion and recollapse.

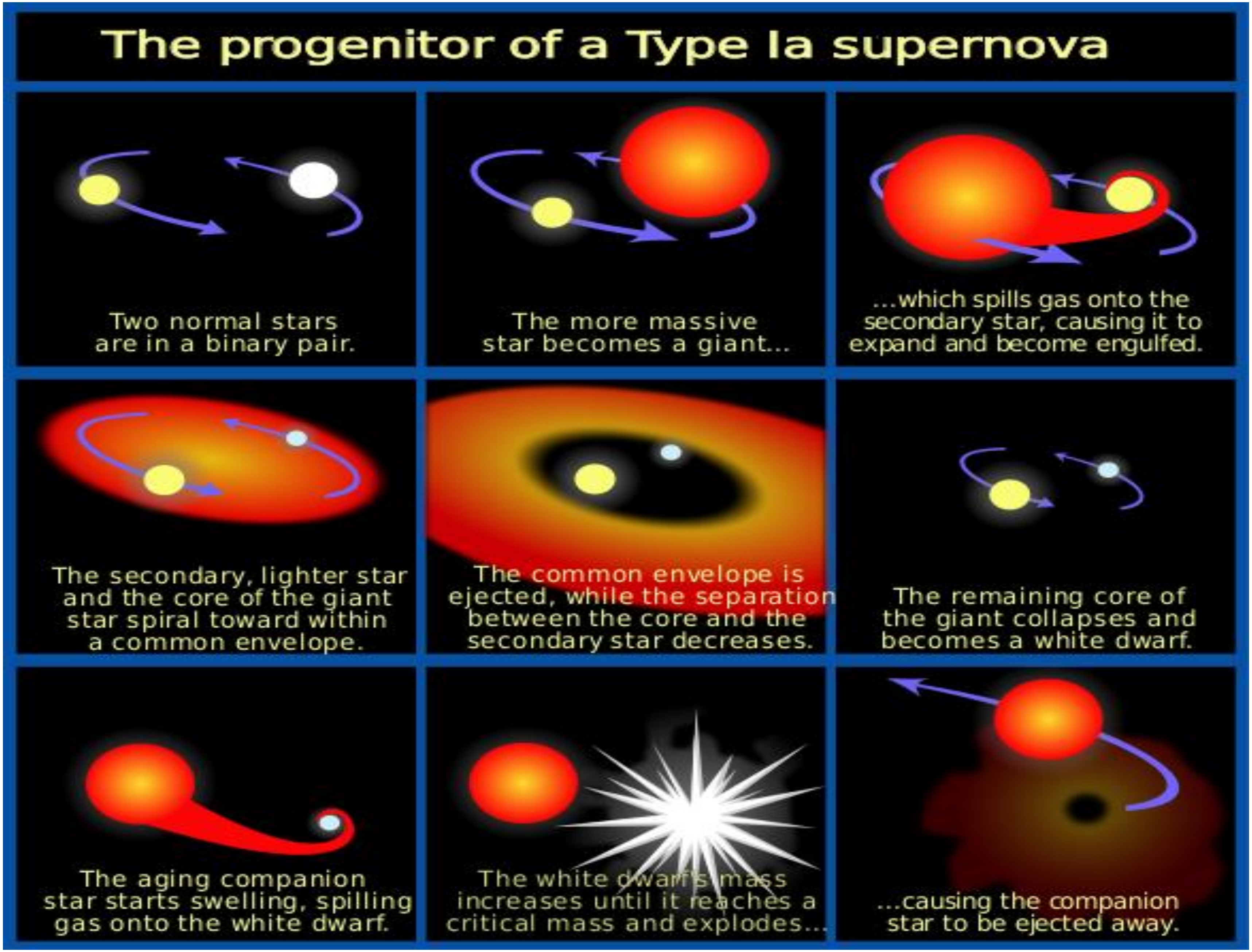
Supernovae Type IA (SNe IA) as Standard Candles.

• SNe IA are uniform in absolute luminosity (dispersion at peak of 1.1 mag): they are suitable as extragalactic distance indicators (Baade W., A0J,88, pag. 285, 1938)

At the present the hypothesis that SNe IA´s are standard candles drew support from:

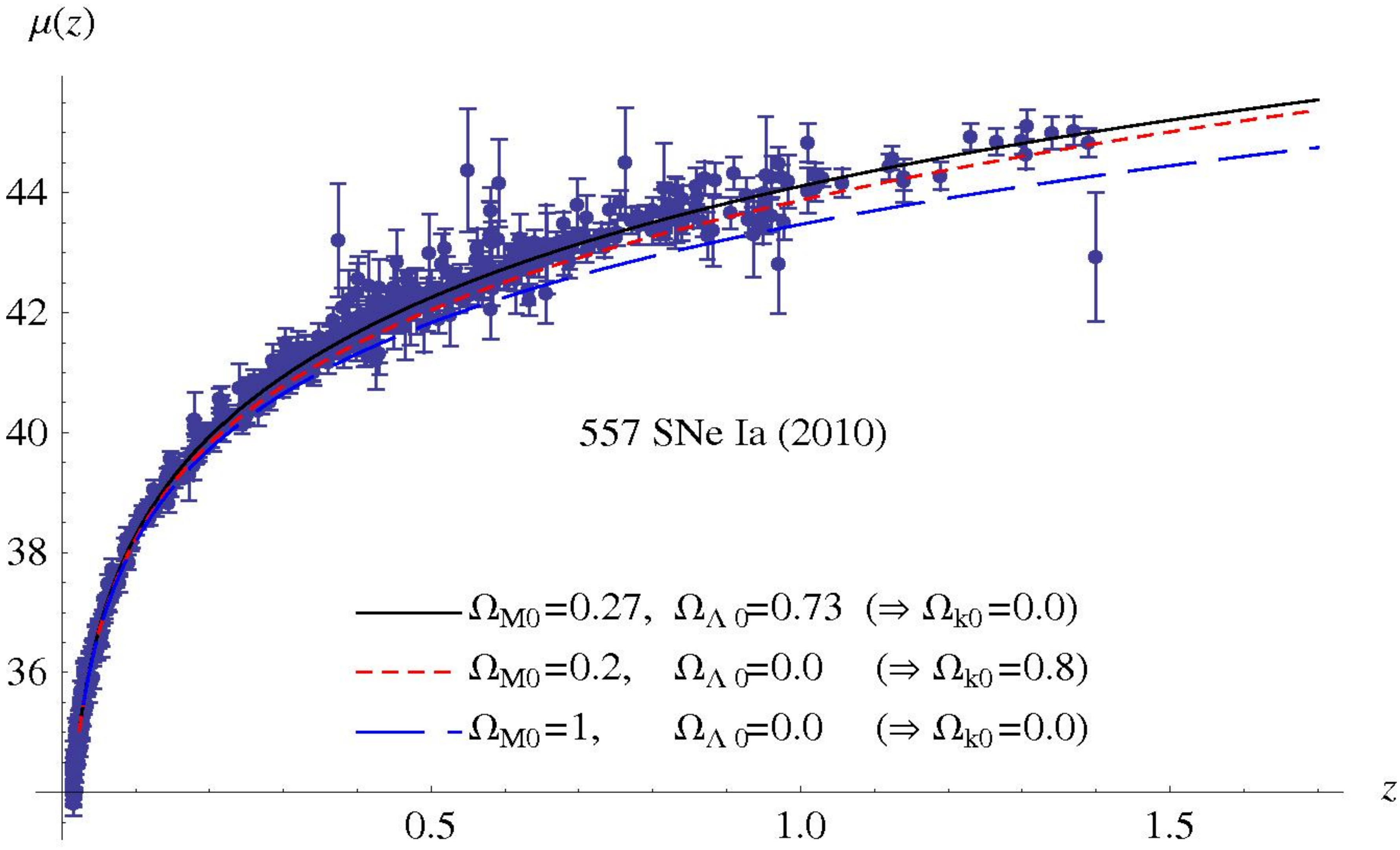
• Empirical Studies: Method using Multicolor Light-Curve Shapes (MLCSs) determining an empirical correlation between the MLCSs and the luminosity of Sne IA´s (A. Riess, W. Press and R. P. Kirshner, ApJ, 88, 473, 1996), (Riess A., et al., Astro-ph/0611572 (2006): The Gold-2006 data.), (Saurabh Jha, A. Riess, R. P. Kirshner, Submitted to ApJ). (dispersion at peak of 0.12 mag: with absolute magnitude M(visible)= -19.44).

• Theoretical Models: these suggested that they arise from ignition of a Carbon-Oxygen white dwarf reaching the Chandrasekhar mass from the acretion of gas and matter of a partner star (like a red giant) leading to a homogeneous light curve and uniform luminosity (Hoyle F. and Fowler W., ApJ, 132, 565, 1960), (Arnett, W., Ap&SS, 5, 280, 1969), (Colgate S. and McKee W., ApJ, 157, 623, 1969). They don´t have hydrogen lines in the spectra.


The progenitor of a Type Ia supernova
Two normal stars are in a binary pair.
The more massive star becomes a giant...
...which spills gas onto the secondary star, causing it to expand and become engulfed.
The secondary, lighter star and the core of the giant star spiral toward within a common envelope.
The common envelope is ejected, while the separation between the core and the secondary star decreases.
The remaining core of the giant collapses and becomes a white dwarf.
The aging companion star starts swelling, spilling gas onto the white dwarf.
The white dwarf's mass increases until it reaches a critical mass and explodes...
...causing the companion star to be ejected away.

Hubble Diagram (Union2 Sample, Amanullah et. al., 2010).

$$\mu(z) \equiv m(z) - M$$



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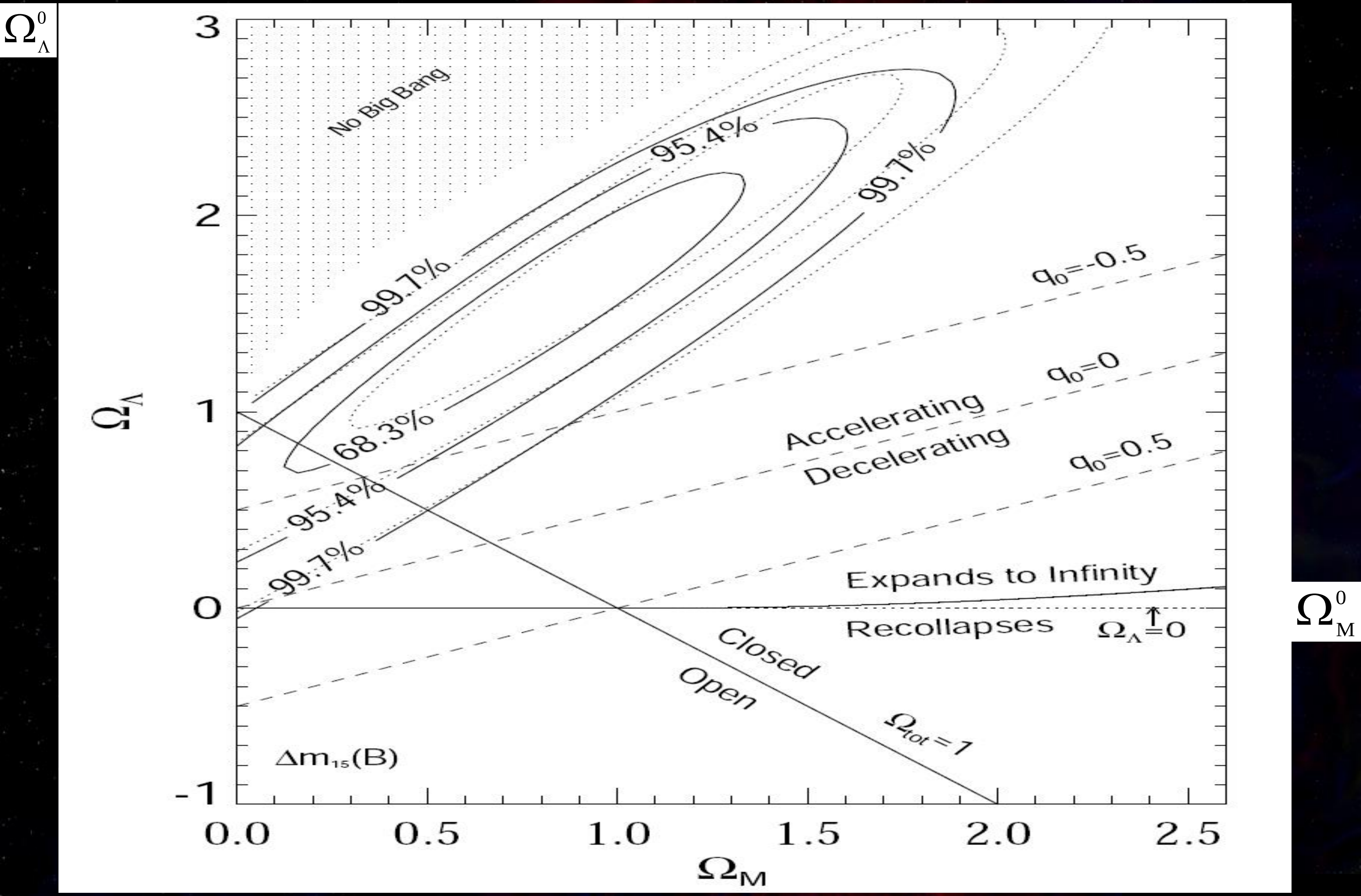
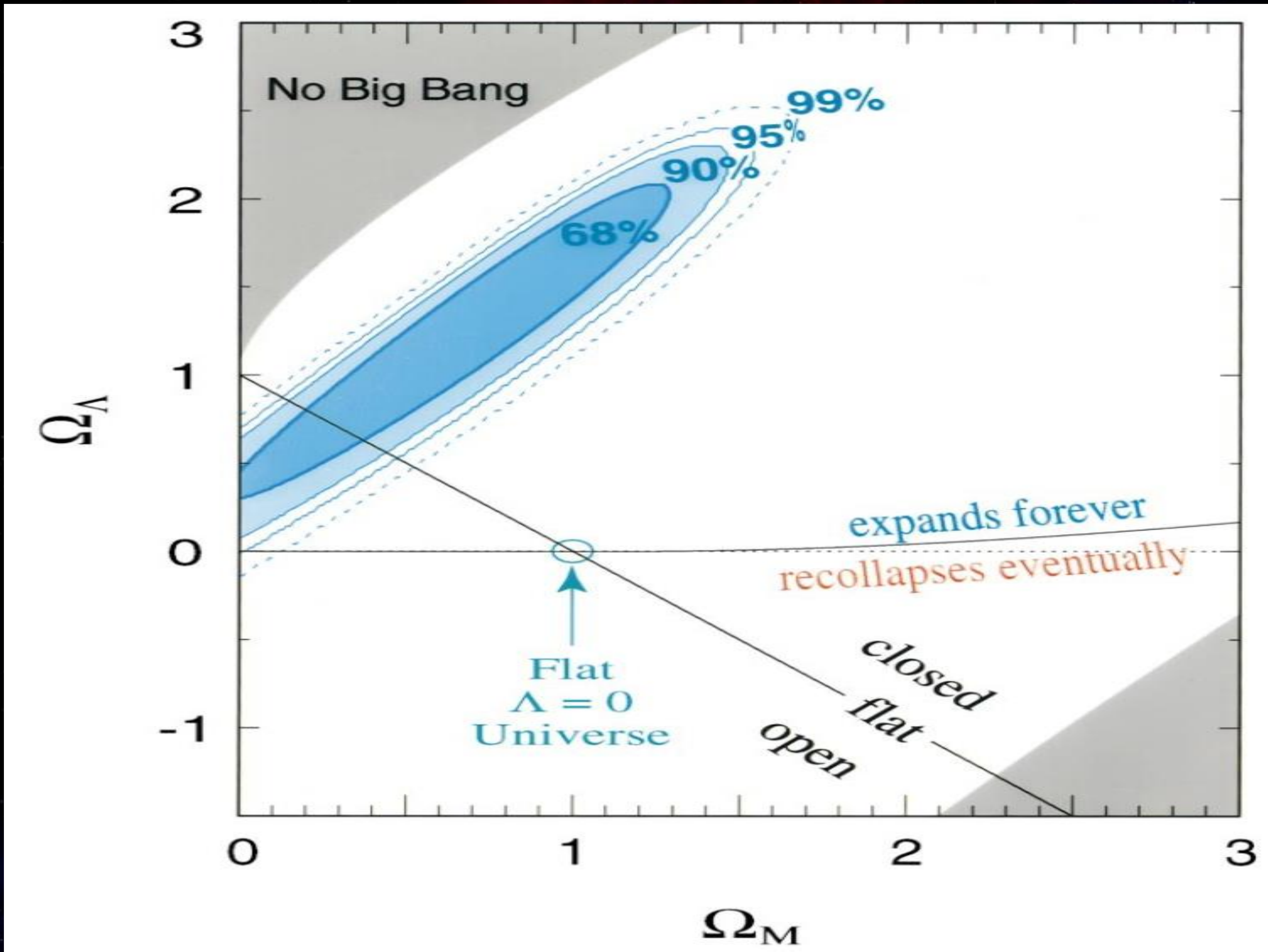
Dividing curve between perpetual expansion and recollapse.

Sne IA Gold Data 2004 and Sne IA Data 1998 (Riess et al.)



SNe Ia Data 1999 (Perlmutter et al.)



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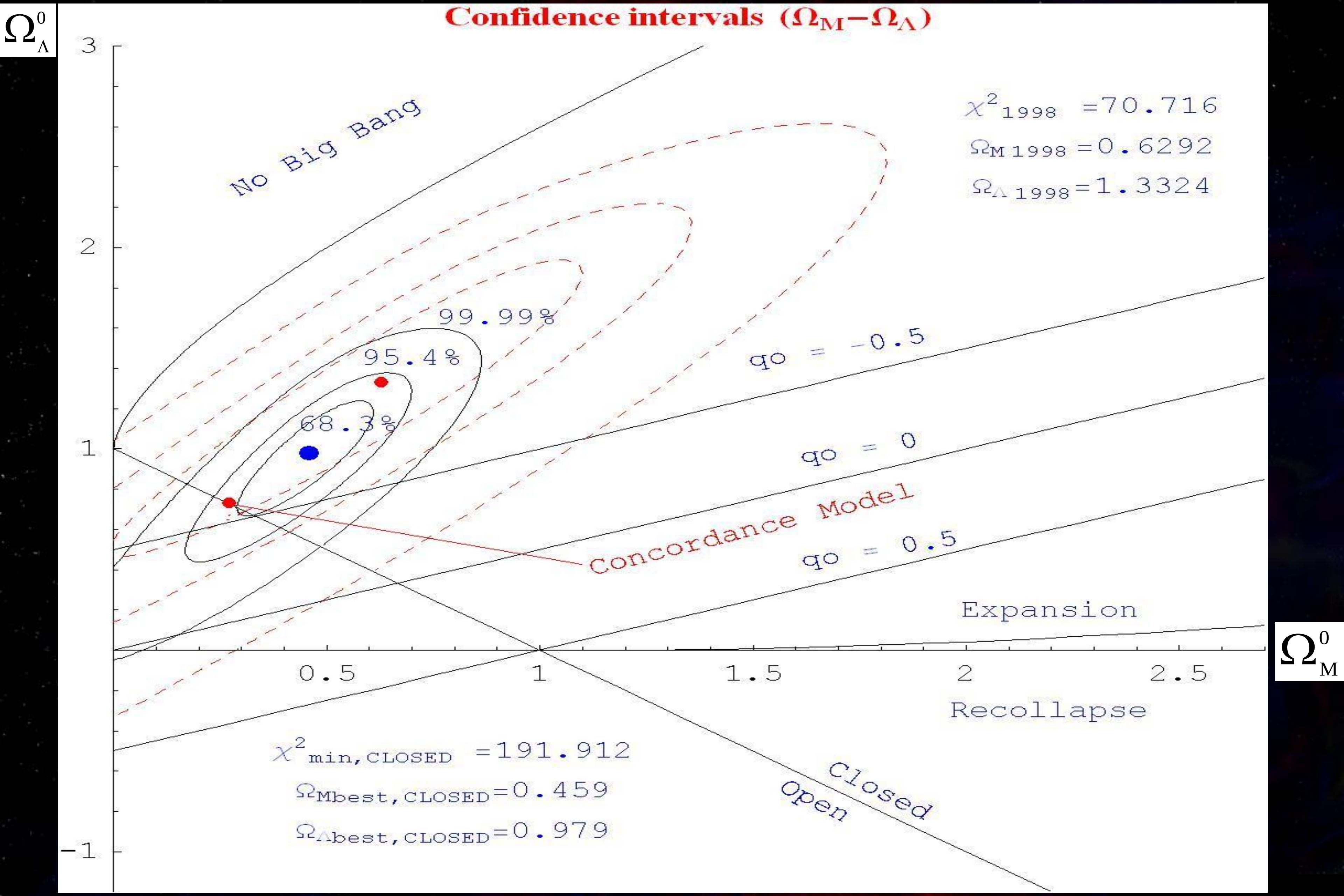


Constraints on for CMB parameters

$$l_A(z_\star) \equiv (1+z_\star)\frac{\pi D_A(z_\star)}{r_s(z_\star)}\,,$$

Acoustic Scale

$$R(z_\star) \equiv \frac{\sqrt{\Omega_M H_0^2}}{c}(1+z_\star)D_A(z_\star)\,.$$

Shift Parameter

$z_\star$

Redshift of Decoupling at last scattering.

Where we are defined:

$$S_k \equiv (1+z)D_{\mathrm{A}}\,,$$

$$S_k(z) = \frac{c}{H_0}\begin{cases} |\Omega_{\mathrm{k}}|^{-1/2}\sinh[\sqrt{\Omega_{\mathrm{k}}}H_0\,r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} > 0), \\ H_0\,r_{\mathrm{c}}(z)/c & \text{if } (\Omega_{\mathrm{k}} = 0), \\ |\Omega_{\mathrm{k}}|^{-1/2}\sin[\sqrt{-\Omega_{\mathrm{k}}}H_0\,r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} < 0), \end{cases}$$

Constraints on for CMB parameters

WMAP Distance Priors Obtained from the *WMAP* Seven-year Fit to Models with Spatial Curvature and Dark Energy

d_i	Seven-year ML[a]	Seven-year Mean[b]	Error, σ
l_A	302.09	302.69	0.76
R	1.725	1.726	0.018
z_*	1091.3	1091.36	0.91

Notes. The correlation coefficients are $r_{l_A,R} = 0.1956$, $r_{l_A,z_*} = 0.4595$, and $r_{R,z_*} = 0.7357$.

[a] Maximum likelihood values (recommended).

[b] Mean of the likelihood.

We compute the Chi-square function:

$$\chi^2_{\rm CMB} = -2\ln L = \sum_{ij}(x_i - d_i)(C^{-1})_{ij}(x_j - d_j),$$

where $x_i = (l_A, R, z_*)$ ← The values predicted by a model

$d_i = (l_A^{WMAP}, R^{WMAP}, z_*^{WMAP})$ ← The data given in the above table

C_{ij}^{-1} ← Covariance Matrix

Inverse Covariance Matrix for the *WMAP* Distance Priors

	l_A	R	z_*
l_A	2.305	29.698	−1.333
R		6825.270	−113.180
z_*			3.414

Baryon Acoustic Oscillation *A*

For a curved universe we have:

$$A \equiv \sqrt{\Omega_m^0}\, E(z_{\mathrm{BAO}})^{-1/3} \left(\frac{1}{z_{\mathrm{BAO}} \sqrt{\left|\Omega_k^0\right|}} \operatorname{Sinn}\left(\sqrt{\left|\Omega_k^0\right|} \int_0^{z_{\mathrm{BAO}}} \frac{dz'}{E(z')} \right) \right)^{2/3}$$

where $$E(z) \equiv \frac{H(z, \Omega_m, \Omega_\Lambda)}{H_0}$$

$z_{\mathrm{BAO}} = 0.35$

χ^2 function

$$\boxed{\chi^2_{\mathrm{BAO}} = \left(\frac{A_{\mathrm{theory}}(\Omega_m, \Omega_\Lambda) - A_{\mathrm{obs}}}{\sigma_A} \right)^2}$$

$A_{\mathrm{observed}} = 0.469 \pm 0.017$

The total χ^2-function

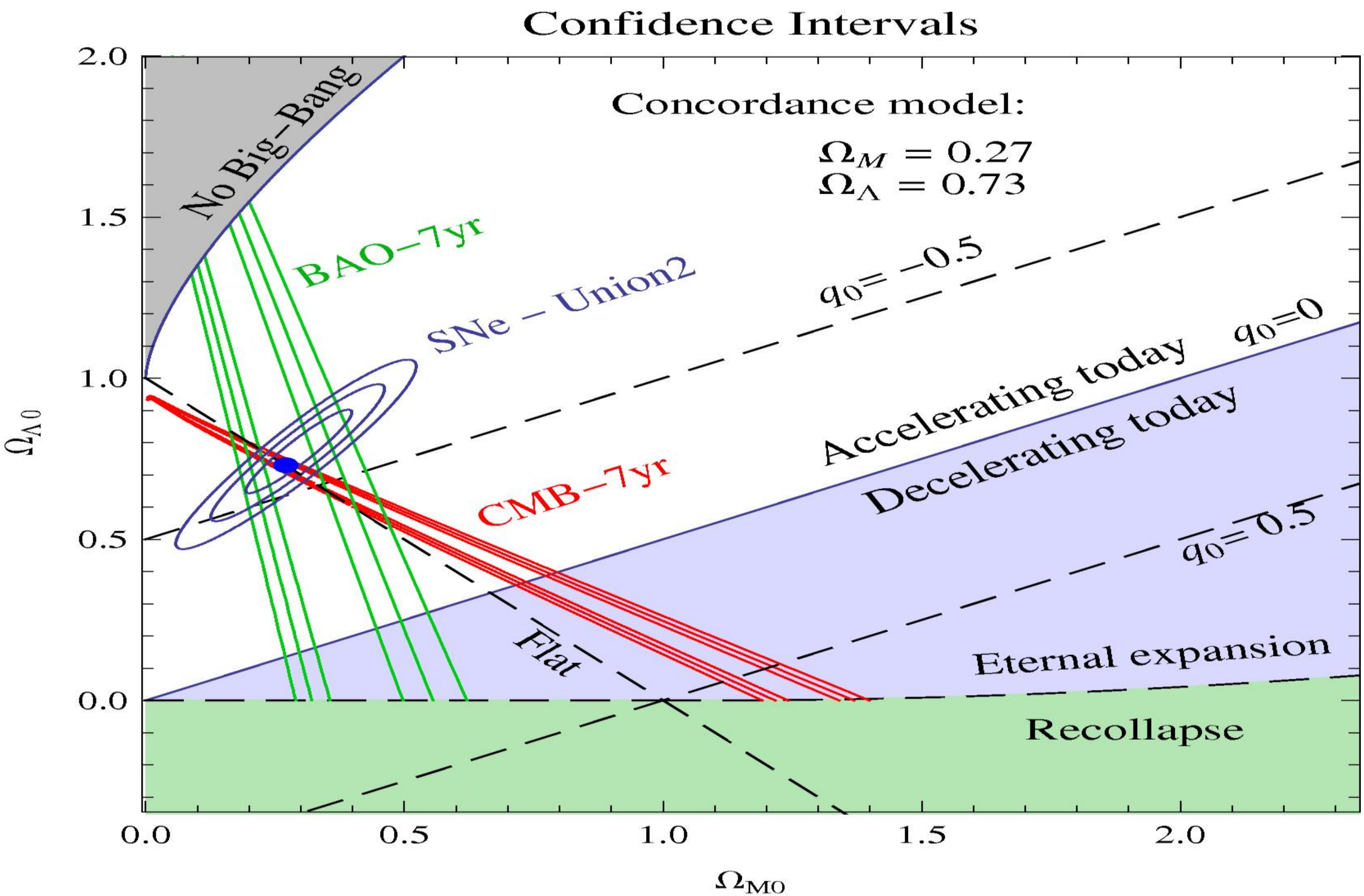
$$\chi^2_{\mathrm{SNe}}(z,\mathbf{X}) \equiv \sum_{k=1}^{n} \frac{[\mu_k^{\mathrm{theory}}(z,\mathbf{X}) - \mu_k^{\mathrm{observ}}]^2}{\sigma_k^2}$$

χ^2_{CMB}

χ^2_{BAO}

$$\chi^2 = \chi^2_{\mathrm{SNe}} + \chi^2_{\mathrm{CMB}} + \chi^2_{\mathrm{BAO}}$$

Constraints on Cosmological Constant and Matter Density Parameters.



Posterior Probability density marginalized on the Hubble constant for universe dominated by matter (dark and baryonic), cosmological constant and curvature:

$$P(\Omega_M^0,\Omega_\Lambda^0)\equiv B\cdot\exp\left[-\frac{\chi^2(\Omega_M^0,\Omega_\Lambda^0)-\chi^2_{\min}}{2}\right]=A\cdot\int_0^\infty\exp\left[-\frac{\tilde{\chi}^2(H_0,\Omega_M^0,\Omega_\Lambda^0)}{2}\right]dH_0$$

We build the Posterior Probability density marginalizing with the prior probability density for flat case:

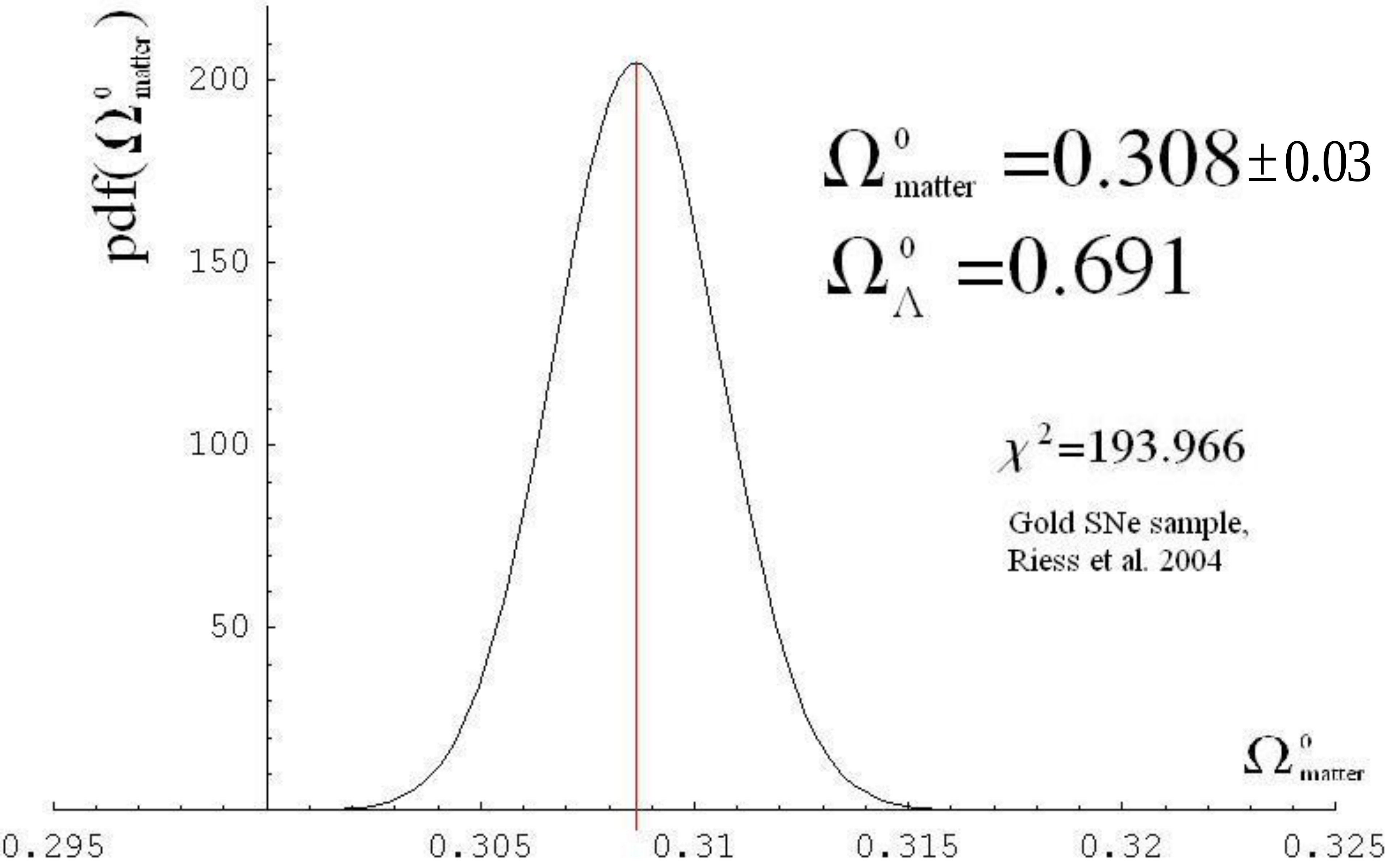
$$\Omega_M+\Omega_\Lambda=1$$

$$\tilde{P}(\Omega_M^0)=D\cdot\exp\left[-\frac{\chi_*^2(\Omega_M^0)-\chi^2_{*\min}}{2}\right]\equiv B\cdot\int_{-\infty}^{\infty}\exp\left[-\frac{\chi^2(\Omega_M^0,\Omega_\Lambda^0)-\chi^2_{\min}}{2}\right]\delta(\Omega_M^0+\Omega_\Lambda^0-1)\,d\Omega_\Lambda^0$$

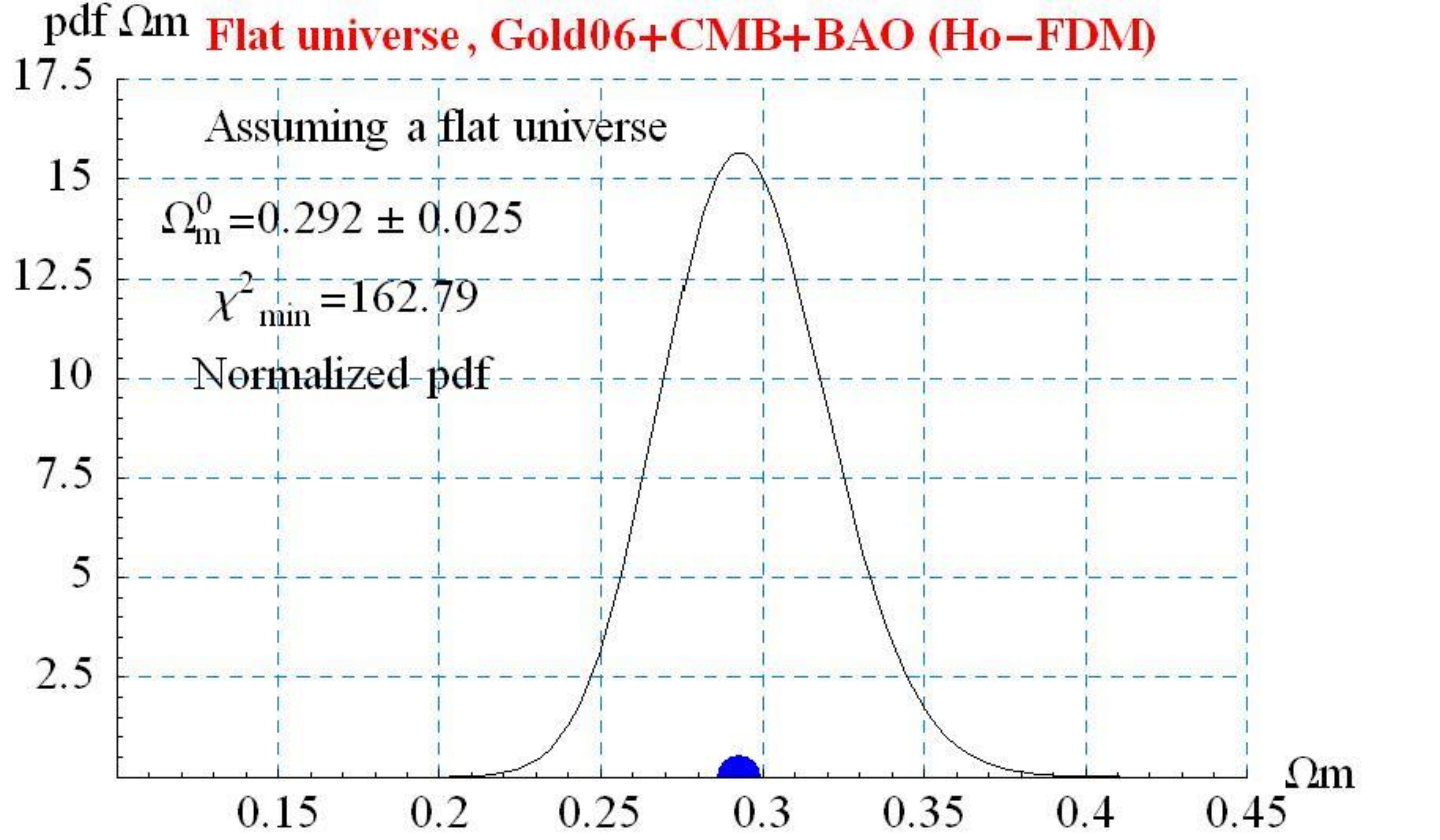
Posterior probability density.

prior probability density for flat case.

Posterior Probability density for the parameter of matter density with the flat prior probability density:



Posterior Probability density for the parameter of matter density with the flat prior probability density:



A flat universe dominated by matter (dust) and a generalized dark energy fluid parameterized by an equation of state with w constant:

$$P_{DE} = c^2 w \rho_{DE}$$

$$P_M \cong 0$$

Posterior Probability density marginalized on the Hubble constant:

$$P(\Omega_M^0, w) \equiv B \cdot \exp\left[-\frac{\chi^2(\Omega_M^0, w) - \chi^2_{\min}}{2}\right] = A \cdot \int_0^\infty \exp\left[-\frac{\tilde{\chi}^2(H_0, \Omega_M^0, w)}{2}\right] dH_0$$

Posterior Probability density with the Gaussian prior probability density for the density parameter of matter coming from anisotropies of CMB or dynamical observations:

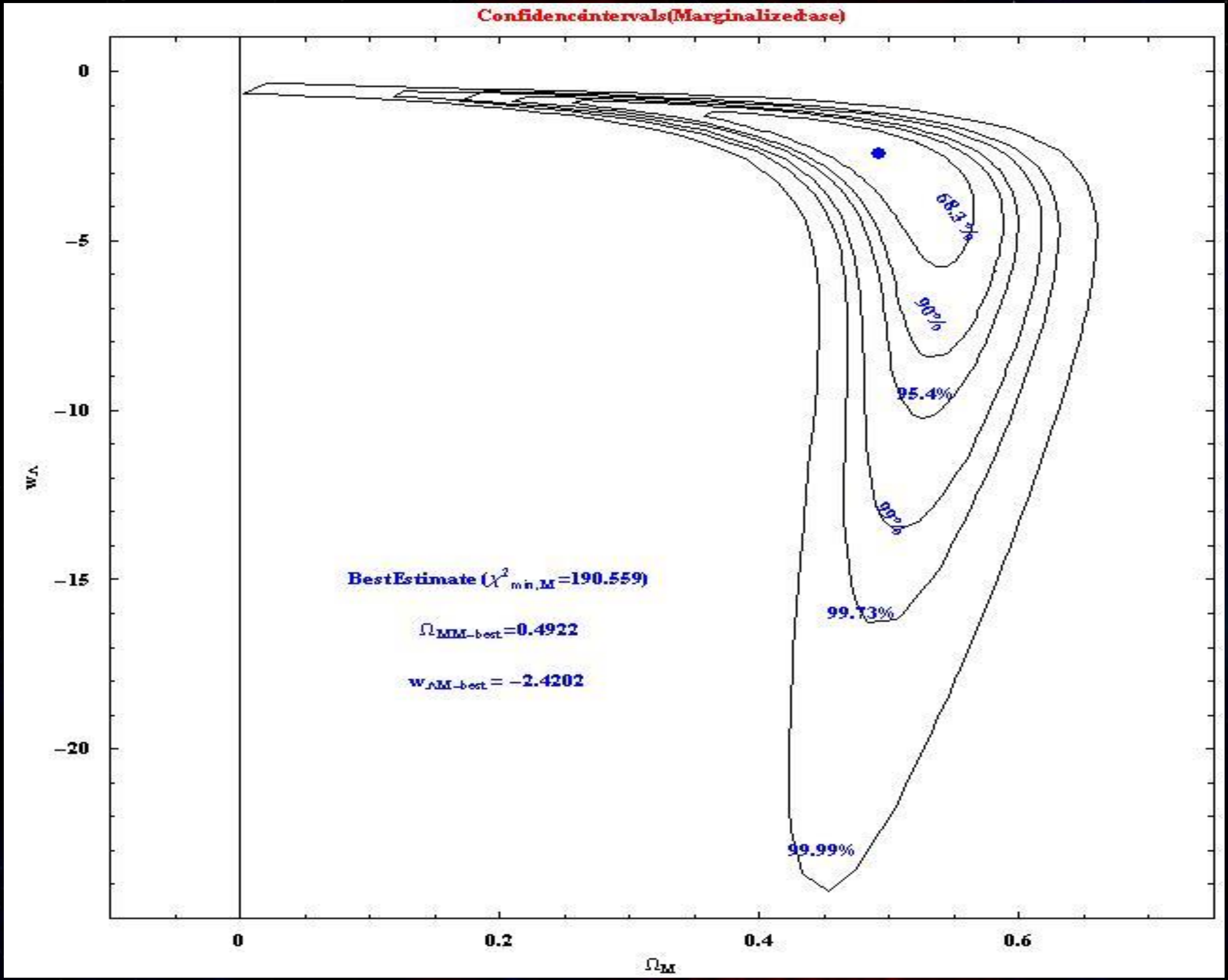
$$\tilde{P}(\Omega_M^0, w) = B \cdot \exp\left[-\frac{\chi^2(\Omega_M^0, w)}{2}\right] \cdot \exp\left(-\frac{(\Omega_M^0 - 0.27)^2}{2(0.04)^2}\right)$$

$$= D \cdot \exp\left[-\frac{\tilde{\chi}^2(\Omega_M^0, w) - \tilde{\chi}^2_{\min}}{2}\right]$$

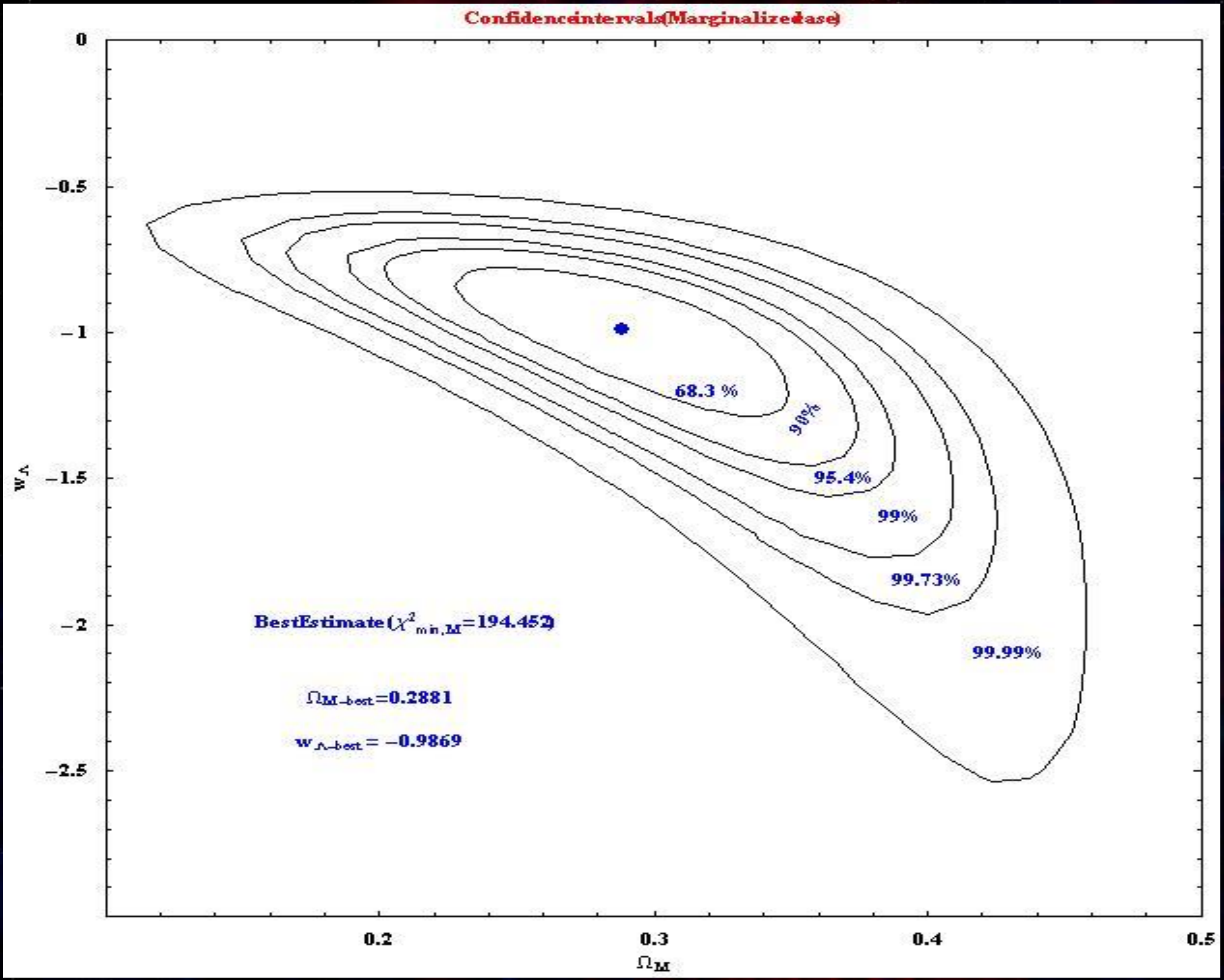
Posterior probability density.

Gaussian prior probability density.

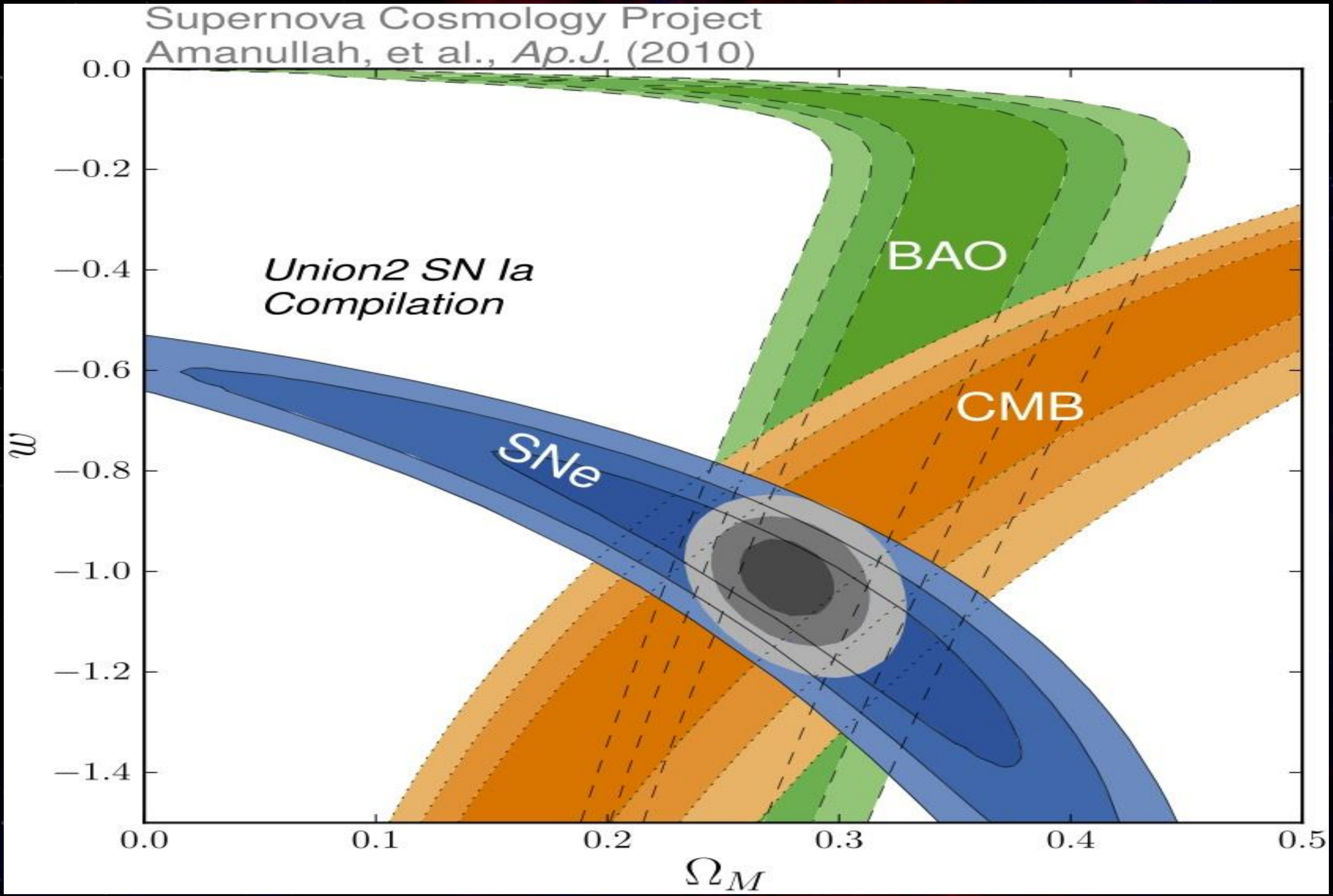
No prior Gaussian for the density of matter

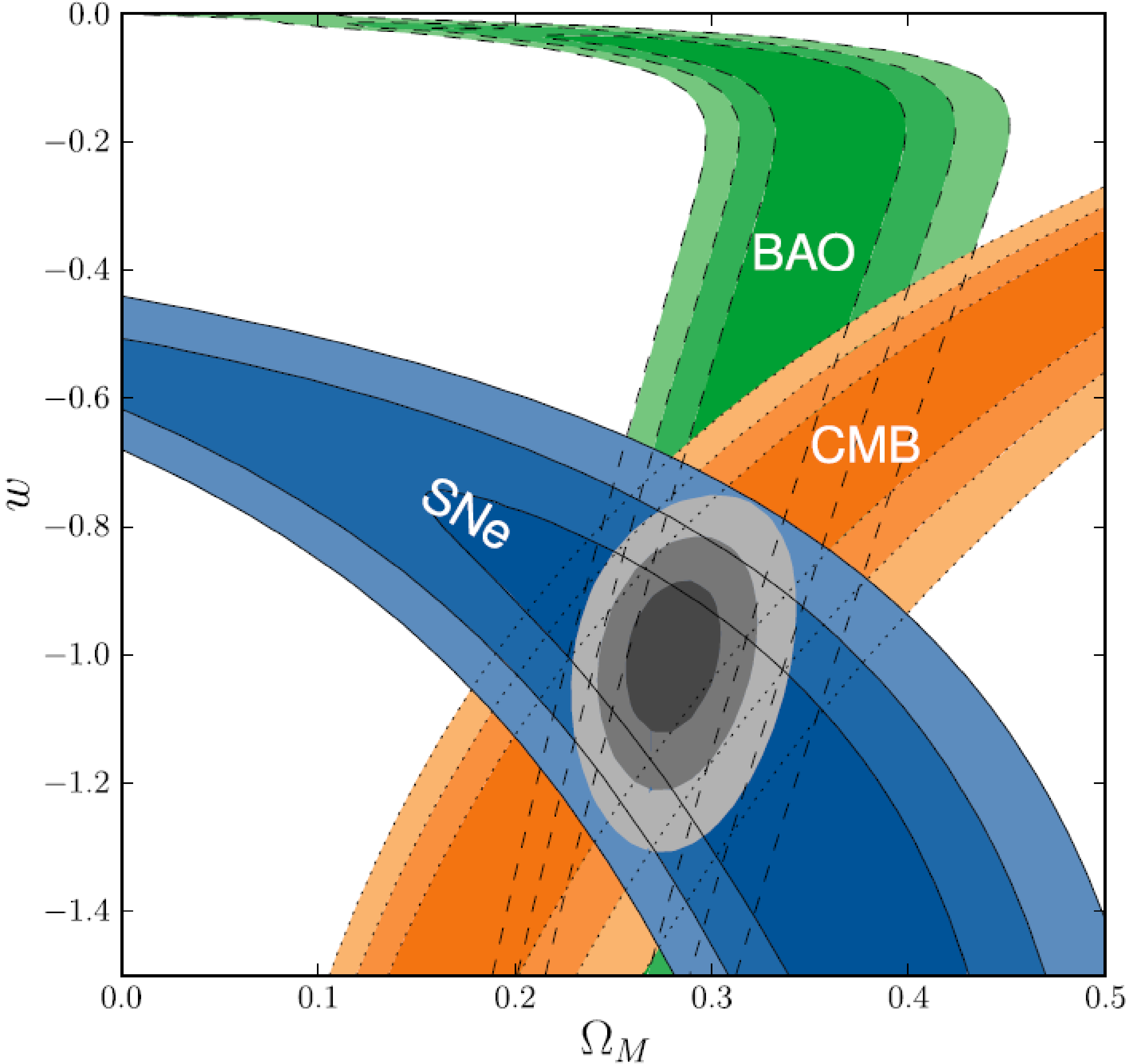


Gaussian prior for density of matter: **!!! It is important to measure with precision the density of barionic and dark matter !!!**



Flat Model (zero curvature) with three combined test: Sne, BAO, CMB.
Confidence regions with w constant. Including systematic errors.




BAO
CMB
SNe
w
Ω_M
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
0.0
0.1
0.2
0.3
0.4
0.5

A flat universe dominated by matter (dust) and a generalized dark energy fluid parameterized by an equation of state with w(z) a lineal function with redshift:

$$P_{DE} = c^2(\mathrm{w}_0 + \mathrm{W}_0 z)\,\rho_{DE}$$

$$P_M \simeq 0$$

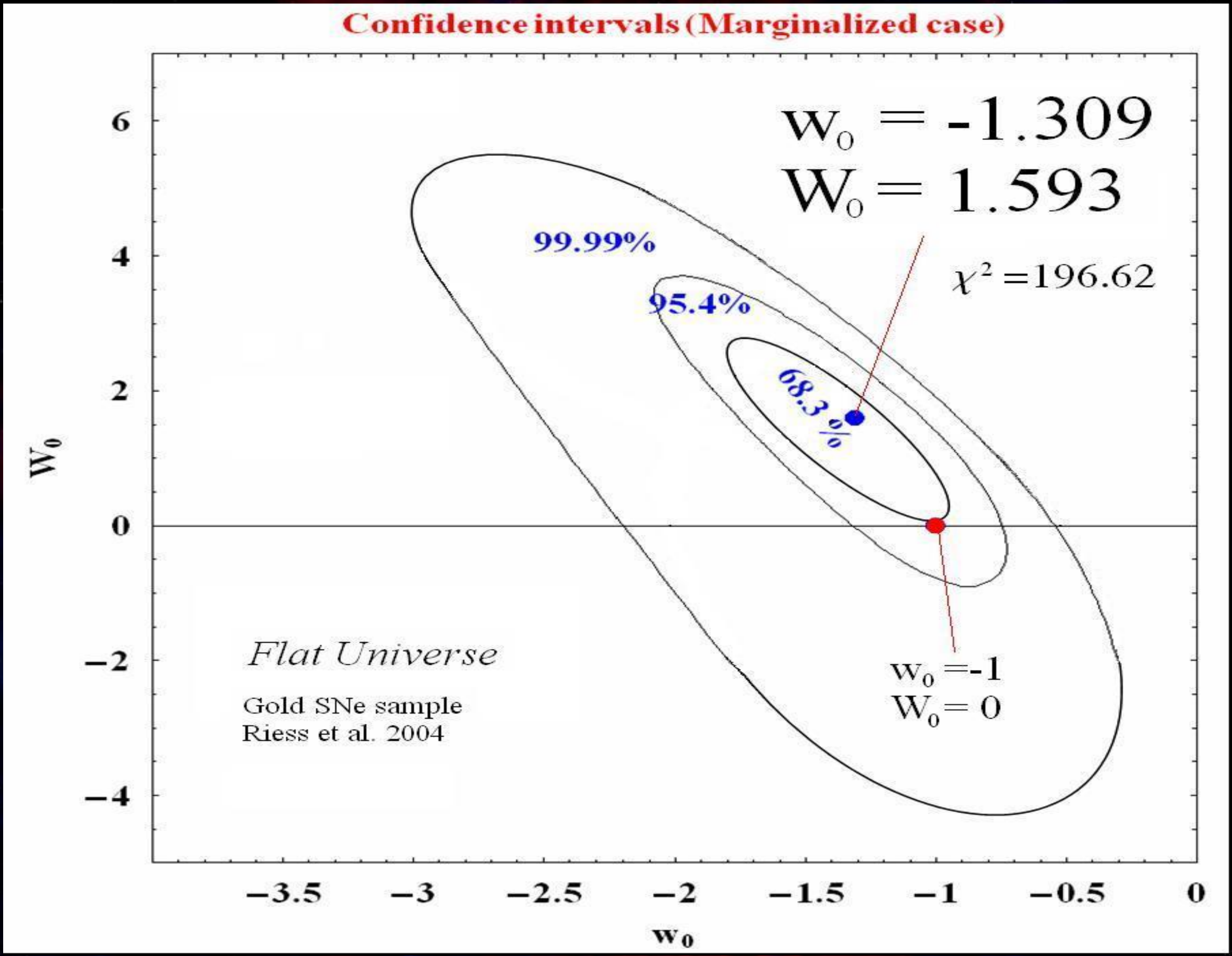
Posterior Probability density marginalized on the Hubble constant:

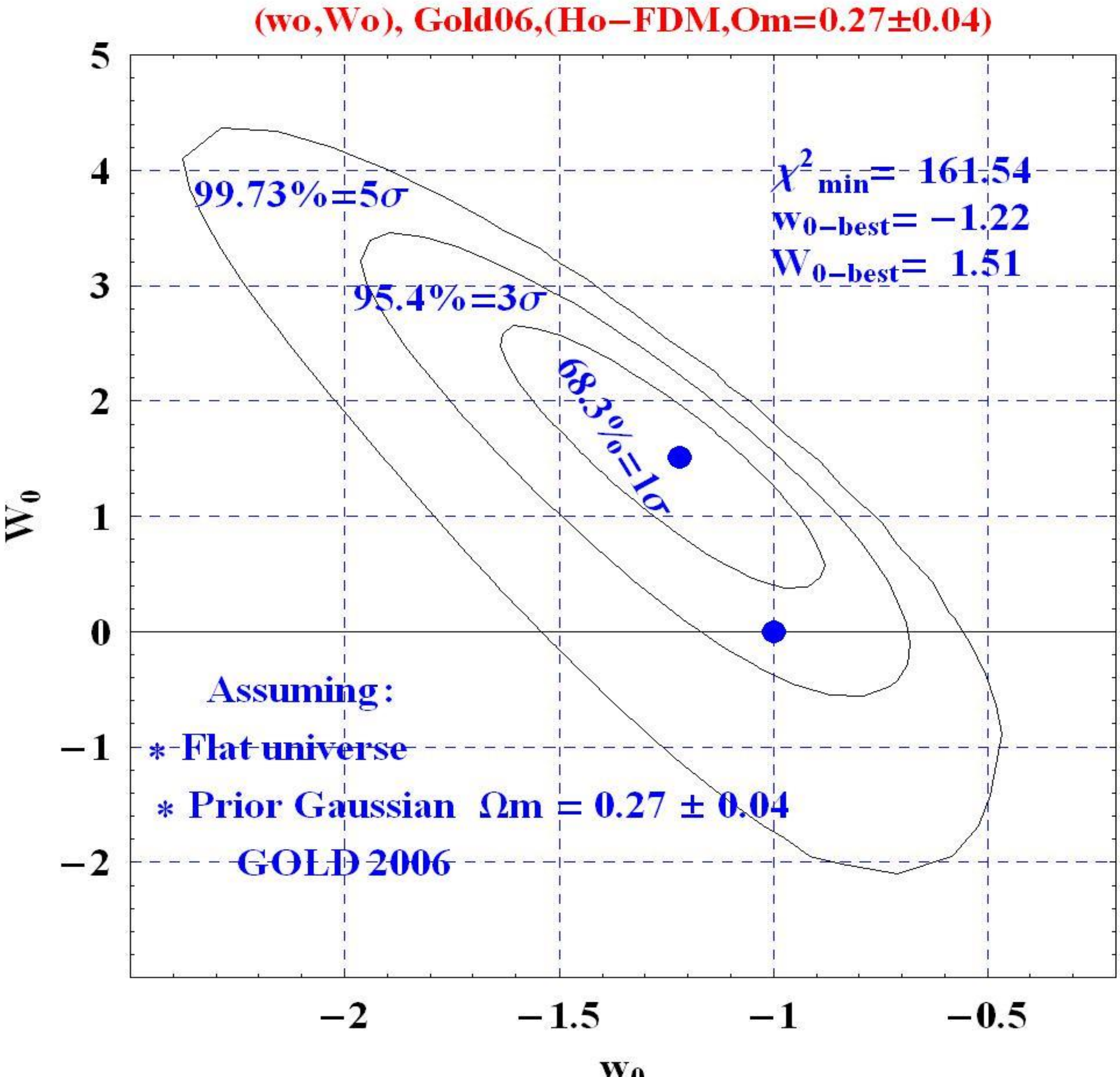
$$P(\Omega_M^0, \mathrm{w}_0, \mathrm{W}_0) \equiv B\exp\left[-\frac{\chi^2(\Omega_M^0, \mathrm{w}_0, \mathrm{W}_0) - \chi^2_{\min}}{2}\right] = A\int_0^\infty \exp\left[-\frac{\tilde{\chi}^2(H_0, \Omega_M^0, \mathrm{w}_0, \mathrm{W}_0)}{2}\right] dH_0$$

Posterior Probability density with the Gaussian prior probability density for the parameter of matter density coming from anisotropies of CMB:

$$\tilde{P}(\mathrm{w}_0, \mathrm{W}_0) = \int_0^\infty B\exp\left[-\frac{\chi^2(\Omega_M^0, \mathrm{w}_0, \mathrm{W}_0)}{2}\right] \exp\left(-\frac{(\Omega_M^0 - 0.27)^2}{2(0.04)^2}\right) d\,\Omega_M^0$$

Gaussian prior for density of matter: **!!! It is important to measure with precision the density of barionic and dark matter !!!**




(wo,Wo), Gold06,(Ho−FDM,Om=0.27±0.04)
χ^2_{min}= 161.54
w_{0-best}= −1.22
W_{0-best}= 1.51
99.73%=5σ
95.4%=3σ
68.3%=1σ
Assuming:
* Flat universe
* Prior Gaussian Ωm = 0.27 ± 0.04
GOLD 2006
W_0
w_0
5
4
3
2
1
0
−1
−2
−2
−1.5
−1
−0.5

Flat Model (zero curvature) with three combined test: Sne, BAO, CMB.
Confidence regions with w constant. Including systematic errors.

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

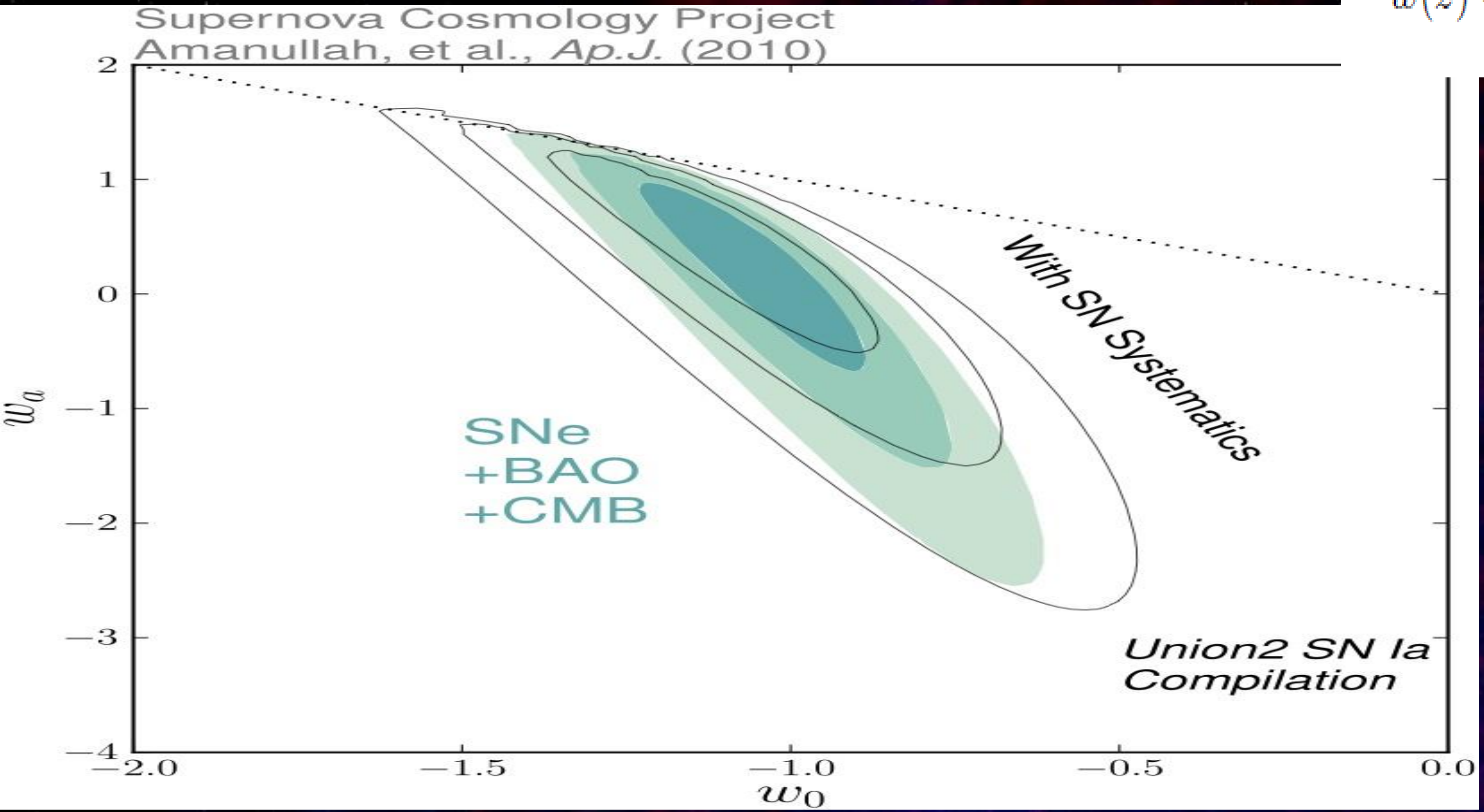


Figure 13. 68.3%, 95.4%, and 99.7% confidence regions of the (w_0, w_a) plane from SNe combined with the constraints from BAO and CMB both with (solid contours) and without (shaded contours) systematic errors. Zero curvature has been assumed. Points above the dotted line ($w_0 + w_a = 0$) violate early matter domination and are implicitly disfavored in this analysis by the CMB and BAO data.

Results with three combined test: Sne, BAO, CMB.

Fit results on cosmological parameters Ω_M, w and Ω_k. The parameter values are followed by their statistical (first column) and statistical and systematic (second column) uncertainties.

Fit	Ω_M	Ω_M w/ Sys	Ω_k	Ω_k w/ Sys	w	w w/ Sys
SNe	$0.270^{+0.021}_{-0.021}$	$0.274^{+0.040}_{-0.037}$	0 (fixed)	0 (fixed)	-1 (fixed)	-1 (fixed)
SNe+BAO+H_0	$0.309^{+0.032}_{-0.032}$	$0.316^{+0.036}_{-0.035}$	0 (fixed)	0 (fixed)	$-1.114^{+0.098}_{-0.112}$	$-1.154^{+0.131}_{-0.150}$
SNe+CMB	$0.268^{+0.019}_{-0.017}$	$0.269^{+0.023}_{-0.022}$	0 (fixed)	0 (fixed)	$-0.997^{+0.050}_{-0.055}$	$-0.999^{+0.074}_{-0.079}$
SNe+BAO+CMB	$0.277^{+0.014}_{-0.014}$	$0.279^{+0.017}_{-0.016}$	0 (fixed)	0 (fixed)	$-1.009^{+0.050}_{-0.054}$	$-0.997^{+0.077}_{-0.082}$
SNe+BAO+CMB	$0.278^{+0.014}_{-0.014}$	$0.281^{+0.018}_{-0.016}$	$-0.004^{+0.006}_{-0.006}$	$-0.004^{+0.006}_{-0.007}$	-1 (fixed)	-1 (fixed)
SNe+BAO+CMB	$0.281^{+0.016}_{-0.015}$	$0.281^{+0.018}_{-0.016}$	$-0.005^{+0.007}_{-0.007}$	$-0.006^{+0.008}_{-0.007}$	$-1.026^{+0.055}_{-0.059}$	$-1.035^{+0.093}_{-0.097}$

Composition of the universe: Concordance Model.

	Concordance Model
•Barionic Matter:	2-5 %
•Dark Matter:	25-30 %
•Electromagnetic Radiation	0.005 %
•Dark Energy	73 %

•Another Components (neutrinos, electrons) ≈ 0 %

$$\rho^{0}_{critica} = 1.88 \times 10^{-29} h^{2} \frac{gr}{cm^{3}}$$

From the quantum vacuum of the Standard Model of particles:

$$\rho_\Lambda^{EW} \approx 3\times10^{47} erg / cm^3$$

$$\rho_\Lambda^{QCD} \approx 1.6\times10^{36} erg / cm^3$$

Planck Density

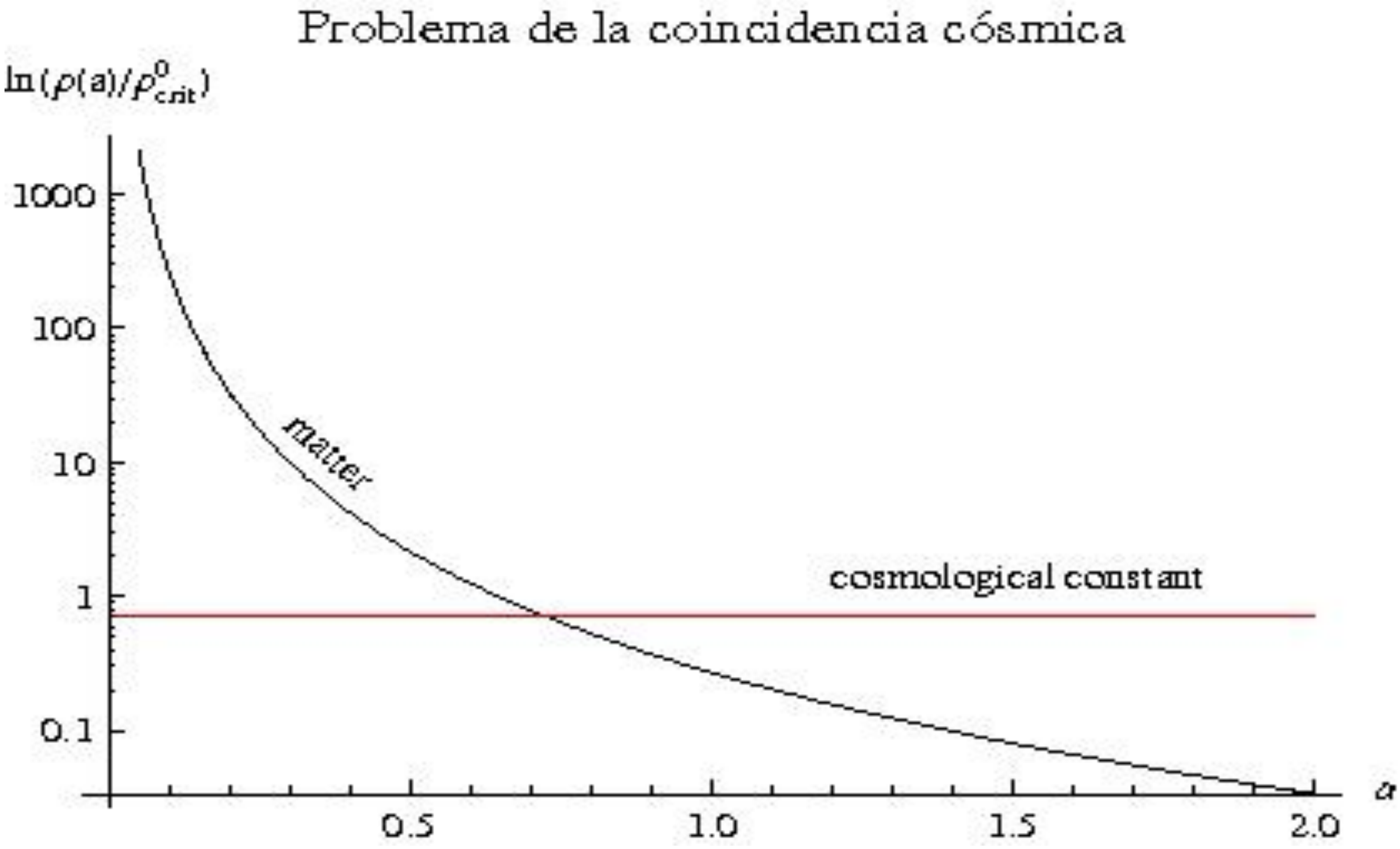
$$\rho_\Lambda^{total} \approx 2\times10^{110} erg / cm^3$$

Density observed for the cosmological constant:

$$\rho_\Lambda^{observada} \approx 2\times10^{-10} erg / cm^3$$

A rate of 120 orders of magnitude !!!

Problema de la Coincidencia Cósmica



Conclusions

Using a kinematic description of the deceleration parameter, the Sne Ia favor recent acceleration and past deceleration at the 99.2% confidence level at the transition redshift :

$$\boxed{z_t = 0.443 \pm 0.14}$$

$$\Omega_M^0 = 0.328 \pm 0.03, \ \Omega_\Lambda^0 = 0.671$$

The SN IA, CMB, BAO samples are consistent with the cosmic concordance model:

$$\boxed{\Omega_M^0 \approx 0.3 , \ \Omega_\Lambda^0 \approx 0.7}$$

Suppose that a survey samples a narrow-redshift shell of width Δz at a redshift z. Furthermore, suppose that we are only interested in the clustering of galaxy pairs with small separations. For a given pair of galaxies, Δz and the angular separation θ are fixed by observation, and we wish to measure the comoving separation for different cosmological models. In the radial direction, separations in comoving space scale with changes in the cosmological model as $\mathrm{d}r_{\mathrm{c}}/\mathrm{d}z \simeq \Delta r_{\mathrm{c}}/\Delta z = c/H(z)$, where $r_{\mathrm{c}}(z) \equiv \int c(1+z)\,\mathrm{d}t$ is the comoving distance to a redshift z. In the angular direction, the comoving galaxy separation scales as $\Delta r_{\mathrm{c}} = \Delta\theta\,(1+z)D_{\mathrm{A}}$, where D_{A} is the standard angular diameter distance. Writing $S_k \equiv (1+z)D_{\mathrm{A}}$,

$$\boxed{a \cdot \Delta r_c = \frac{\Delta r_c}{1+z} = \Delta\theta \cdot D_A} \longrightarrow \boxed{\Delta r_c = \Delta\theta \cdot (1+z) \cdot D_A}$$

Where we have:

$$\boxed{S_k(z) = r^* = \text{ comoving distance.}}$$

$$S_k(z) = \frac{c}{H_0}\begin{cases} |\Omega_{\mathrm{k}}|^{-1/2}\sinh[\sqrt{\Omega_{\mathrm{k}}}H_0\, r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} > 0), \\ H_0\, r_{\mathrm{c}}(z)/c & \text{if } (\Omega_{\mathrm{k}} = 0), \\ |\Omega_{\mathrm{k}}|^{-1/2}\sin[\sqrt{-\Omega_{\mathrm{k}}}H_0\, r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} < 0), \end{cases}$$

Constraints on for CMB parameters

$$l_A(z_\star) \equiv (1+z_\star)\frac{\pi D_A(z_\star)}{r_s(z_\star)},$$

Acoustic Scale

$$R(z_\star) \equiv \frac{\sqrt{\Omega_M H_0^2}}{c}(1+z_\star)D_A(z_\star).$$

Shift Parameter

$z_\star$

Redshift of Decoupling at last scattering.

Where we are defined:

$$S_k \equiv (1+z)D_{\mathrm{A}},$$

$$S_k(z) = \frac{c}{H_0}\begin{cases} |\Omega_{\mathrm{k}}|^{-1/2}\sinh[\sqrt{\Omega_{\mathrm{k}}}H_0\, r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} > 0), \\ H_0\, r_{\mathrm{c}}(z)/c & \text{if } (\Omega_{\mathrm{k}} = 0), \\ |\Omega_{\mathrm{k}}|^{-1/2}\sin[\sqrt{-\Omega_{\mathrm{k}}}H_0\, r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} < 0), \end{cases}$$

Constraints on for CMB parameters

WMAP Distance Priors Obtained from the *WMAP* Seven-year Fit to Models with Spatial Curvature and Dark Energy

d_i	Seven-year ML[a]	Seven-year Mean[b]	Error, σ
l_A	302.09	302.69	0.76
R	1.725	1.726	0.018
z_*	1091.3	1091.36	0.91

Notes. The correlation coefficients are $r_{l_A,R} = 0.1956$, $r_{l_A,z_*} = 0.4595$, and $r_{R,z_*} = 0.7357$.

[a] Maximum likelihood values (recommended).

[b] Mean of the likelihood.

We compute the Chi-square function:

$$\chi^2_{\mathrm{CMB}} = -2\ln L = \sum_{ij}(x_i - d_i)(C^{-1})_{ij}(x_j - d_j),$$

where $x_i = (l_A, R, z_*)$ ← The values predicted by a model

$d_i = (l_A^{WMAP}, R^{WMAP}, z_*^{WMAP})$ ← The data given in the above table

C_{ij}^{-1} ← Covariance Matrix

Inverse Covariance Matrix for the *WMAP* Distance Priors

	l_A	R	z_*
l_A	2.305	29.698	−1.333
R		6825.270	−113.180
z_*			3.414

Constraints on for CMB parameters

$$l_A(z_\star) \equiv (1+z_\star)\frac{\pi D_A(z_\star)}{r_s(z_\star)}\,,$$

Acoustic Scale

$$R(z_\star) \equiv \frac{\sqrt{\Omega_M H_0^2}}{c}(1+z_\star)D_A(z_\star)\,.$$

Shift Parameter

$$z_d = 1291\frac{(\Omega_0 h^2)^{0.251}}{1+0.659(\Omega_0 h^2)^{0.828}}[1+b_1(\Omega_b h^2)^{b}$$

$$b_1 = 0.313(\Omega_0 h^2)^{-0.419}[1+0.607(\Omega_0 h^2)^{0.674}]$$

$$b_2 = 0.238(\Omega_0 h^2)^{0.223}\,,$$

$$S_k \equiv (1+z)D_{\mathrm{A}}\,,$$

$$S_k(z) = \frac{c}{H_0}\begin{cases} |\Omega_{\mathrm{k}}|^{-1/2}\sinh[\sqrt{\Omega_{\mathrm{k}}}H_0\,r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} > 0), \\ H_0\,r_{\mathrm{c}}(z)/c & \text{if } (\Omega_{\mathrm{k}} = 0), \\ |\Omega_{\mathrm{k}}|^{-1/2}\sin[\sqrt{-\Omega_{\mathrm{k}}}H_0\,r_{\mathrm{c}}(z)/c] & \text{if } (\Omega_{\mathrm{k}} < 0), \end{cases}$$

Suppose that a survey samples a narrow-redshift shell of width Δz at a redshift z. Furthermore, suppose that we are only interested in the clustering of galaxy pairs with small separations. For a given pair of galaxies, Δz and the angular separation θ are fixed by observation, and we wish to measure the comoving separation for different cosmological models. In the radial direction, separations in comoving space scale with changes in the cosmological model as $\mathrm{d}r_c/\mathrm{d}z \simeq \Delta r_c/\Delta z = c/H(z)$, where $r_c(z) \equiv \int c(1+z)\,\mathrm{d}t$ is the comoving distance to a redshift z. In the angular direction, the comoving galaxy separation scales as $\Delta r_c = \Delta\theta\,(1+z)D_A$, where D_A is the standard angular diameter distance. Writing $S_k \equiv (1+z)D_A$,

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Baryon Acoustic Oscillation *A*

For a curved universe we have:

$$A \equiv \sqrt{\Omega_m^0}\, E(z_{BAO})^{-1/3} \left(\frac{1}{z_{BAO}\sqrt{|\Omega_k^0|}} \,\mathrm{Sinn}\left(\sqrt{|\Omega_k^0|} \int_0^{z_{BAO}} \frac{dz'}{E(z')} \right) \right)^{2/3}$$

where $E(z) \equiv \dfrac{H(z, \Omega_m, \Omega_\Lambda)}{H_0}$

$z_{BAO} = 0.35$

χ^2 function

$$\boxed{\chi^2_{BAO} = \left(\frac{A_{theory}(\Omega_m, \Omega_\Lambda) - A_{obs}}{\sigma_A} \right)^2}$$

$A_{observed} = 0.469 \pm 0.017$

The total χ^2-function

$$\chi^2_{\mathrm{SNe}}(z,\mathbf{X}) \equiv \sum_{k=1}^{n} \frac{[\mu_k^{\mathrm{theory}}(z,\Omega_{\mathrm{m}},\Omega_{\Lambda})-\mu_k^{\mathrm{observ}}]^2}{\sigma_k^2}$$

$$\chi^2_{\mathrm{BAO}} = \left(\frac{A_{\mathrm{theory}}(\Omega_{\mathrm{m}},\Omega_{\Lambda})-A_{\mathrm{obs}}}{\sigma_A}\right)^2$$

$$\boxed{\chi^2 = \chi^2_{\mathrm{SNe}} + \chi^2_{\mathrm{BAO}}}$$

CMB shift parameter *R*

$$R \equiv \sqrt{\Omega_{\mathrm{m}}^{0}} \int_{0}^{z_{\mathrm{CMB}}} \frac{dz'}{E(z')}$$

where

$$E(z) \equiv \frac{H(z)}{H_0}$$

$z_{\mathrm{CMB}} = 1089$

χ^2 function

$$\chi^2_{\mathrm{CMB}} = \left(\frac{R - R_{\mathrm{obs}}}{\sigma_R} \right)^2$$

$R_{\mathrm{observed}} = 1.70 \pm 0.03$

From the quantum vacuum of the Standard Model of particles:

$$\rho_{\Lambda}^{EW} \approx 3\times10^{47} erg / cm^{3}$$

$$\rho_{\Lambda}^{QCD} \approx 1.6\times10^{36} erg / cm^{3}$$

Planck Density

$$\rho_{\Lambda}^{total} \approx 2\times10^{110} erg / cm^{3}$$

Density observed for the cosmological constant:

$$\rho_{\Lambda}^{observada} \approx 2\times10^{-10} erg / cm^{3}$$

A rate of 120 orders of magnitude !!!

Del vacío cuántico del Modelo Estándar de partículas elementales:

$$\rho_\Lambda^{EW} \approx 3\times 10^{47} erg/cm^3$$

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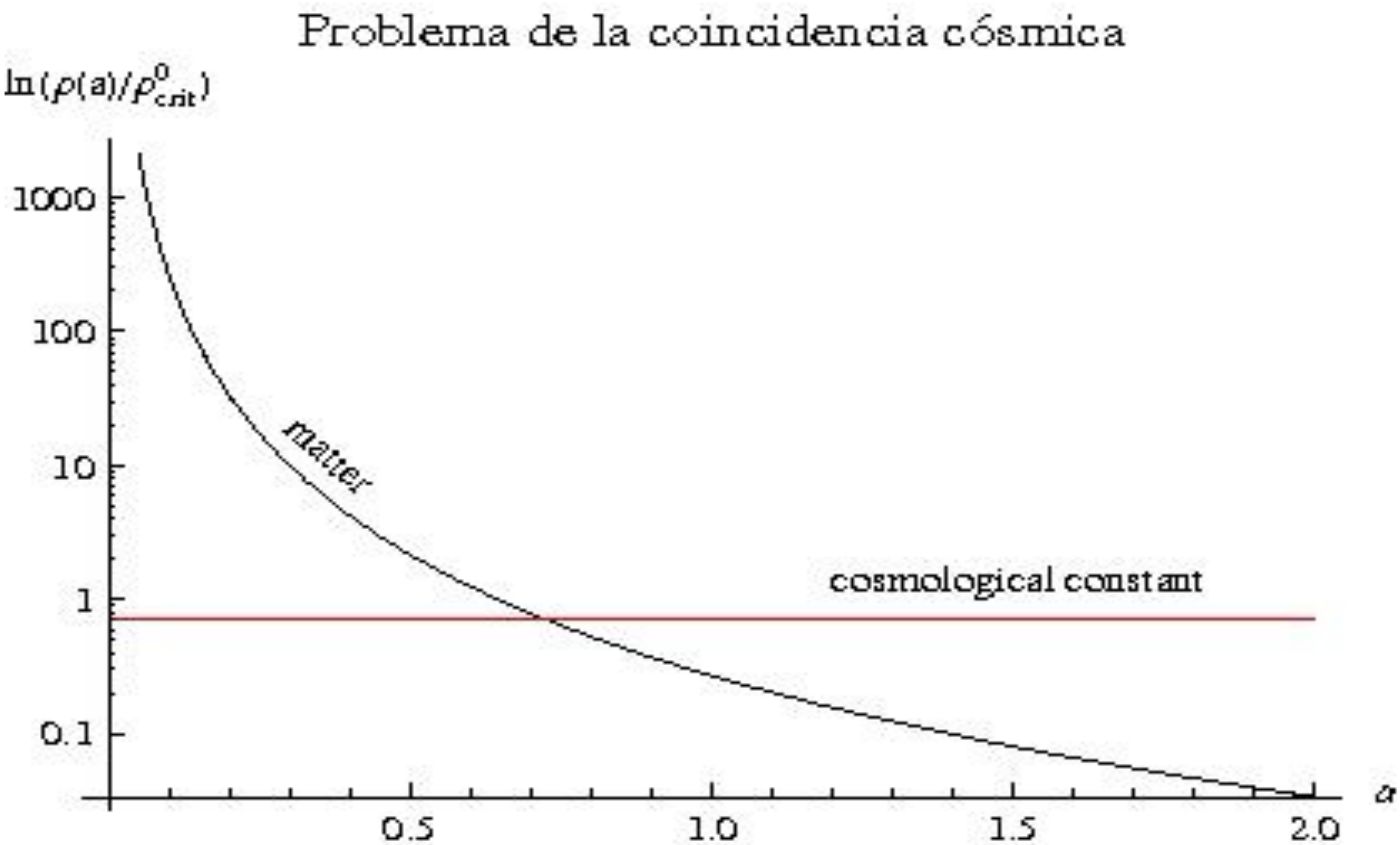
Densidad de Planck

Densidad observada para la constante cosmologica:

$$\rho_\Lambda^{observada} \approx 2\times 10^{-10} erg/cm^3$$

Una razón de 120 órdenes de magnitud !!!

Problema de la Coincidencia Cósmica



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$z_{CMB} = 1089$

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Baryon Acoustic Oscillation *A*

$$A \equiv \sqrt{\Omega_{\mathrm{m}}^{0}}\,E(z_{\mathrm{BAO}})^{-1/3}\left(\frac{1}{z_{\mathrm{BAO}}}\int_{0}^{z_{\mathrm{BAO}}}\frac{dz'}{E(z')}\right)^{2/3}$$

where

$$E(z) \equiv \frac{H(z)}{H_0}$$

$z_{\mathrm{BAO}} = 0.35$

χ^2 function

$$\chi^2_{\mathrm{BAO}} = \left(\frac{A - A_{\mathrm{obs}}}{\sigma_A}\right)^2$$

$A_{\mathrm{observed}} = 0.469 \pm 0.017$

The total χ^2-function

$$\chi^2_{\mathrm{SNe}}(z, \mathbf{X}) \equiv \sum_{k=1}^{n} \frac{[\mu_k^{\mathrm{theory}}(z, \mathbf{X}) - \mu_k^{\mathrm{observ}}]^2}{\sigma_k^2}$$

$$\chi^2_{\mathrm{CMB}} = \left(\frac{R - R_{\mathrm{obs}}}{\sigma_R} \right)^2$$

$$\chi^2_{\mathrm{BAO}} = \left(\frac{A - A_{\mathrm{obs}}}{\sigma_A} \right)^2$$

$$\boxed{\chi^2 = \chi^2_{\mathrm{SNe}} + \chi^2_{\mathrm{CMB}} + \chi^2_{\mathrm{BAO}}}$$

Composition of the universe: Concordance Model.

	Concordance Model
•Barionic Matter:	2-5 %
•Dark Matter:	25-30 %
•Electromagnetic Radiation	0.005 %
•Dark Energy	73 %

•Another Components (neutrinos, electrons) ≈ 0 %

$$\rho^0_{critica} = 1.88 \times 10^{-29} h^2 \frac{gr}{cm^3}$$

Conclusions

Using a lineal kinematic description of the deceleration parameter, the Sne Ia favor recent acceleration and past deceleration at the 99.2% confidence level at the transition redshift :

$$z_t = 0.44233 \pm 0.14$$

The Gold sample 2004 and ¿2006? (with a flat prior probability density) and the SNLS sample are consistent with the cosmic concordance model:

$$\Omega_M^0 \approx 0.3\,,\ \Omega_\Lambda^0 \approx 0.7$$

$$\Omega_M^0 = 0.292 \pm 0.025, \quad \Omega_\Lambda^0 = 0.707 \pm 0.075$$

- For a flat universe with a cosmological constant we measure:

$$\Omega_M^0 = 0.308 \pm 0.03\,,\ \Omega_\Lambda^0 = 0.691$$

Gold Data 2004

$$\Omega_M^0 = 0.342 \pm 0.02\,,\ \Omega_\Lambda^0 = 0.658$$

Gold Data 2006

$$\Omega_M^0 = 0.278 \pm 0.04\,,\ \Omega_\Lambda^0 = 0.721$$

SNLS Data 2006

Modelos propuestos para explicar la aceleración reciente del Universo:

- **Campos Escalares: Quintessence.**
- **Campos Escalares Fantasmas (Phantom Energy).**
- **Fluidos de Chaplygin.**
- **Fluidos Imperfectos Con Viscosidad.**
- **Energía Obscura Holográfica.**
- **Modelos de Neutrinos con Masa Cambiante.**
- **Teorías de Gran Unificación Supersimétricas.**
- **Teorías Alternativas a la Relatividad General:**

1. **Teorías Tensor-Escalares.**
2. **Modificaciones de Curvatura a la Accion de Relatividad General.**

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$$\Omega_M^0 = 0.328 \pm 0.03, \; \Omega_\Lambda^0 = 0.671$$

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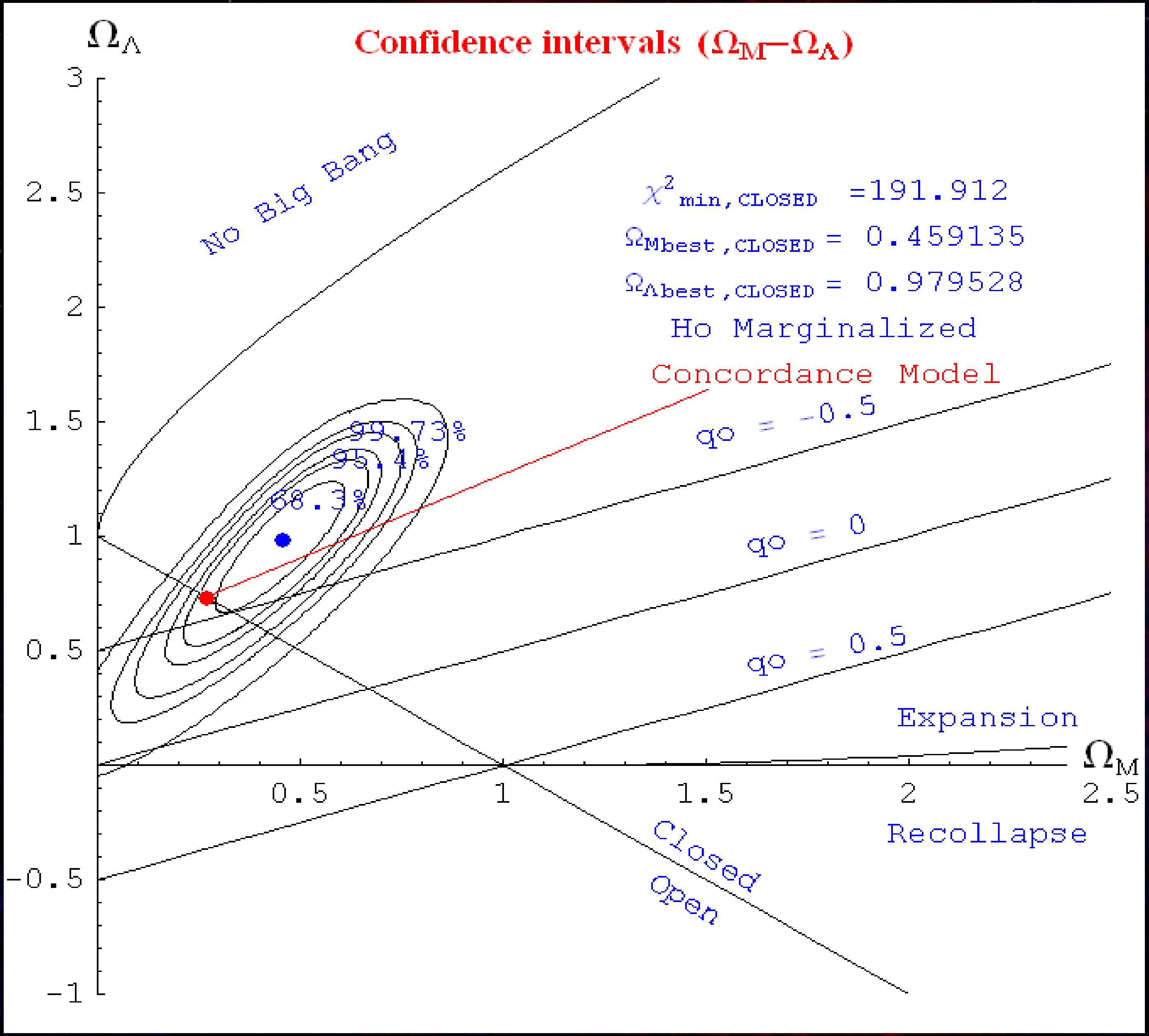
The Gold Data



χ2 of Sne Ia

$$\chi^2(H_0, \mathbf{X}) \equiv \sum_{k=1}^{n} \frac{[\mu^{t}(z_k, H_0, \mathbf{X}) - \mu_k]^2}{\sigma_k^2}$$

The shift parameter R, of the Cosmic Microwave Background radiation

$$R \equiv \sqrt{\Omega_{\rm m}^0} \int_0^{z_{\rm CMB}} \frac{dz'}{E(z')}$$

where z_{CMB} = 1089

R_{obs} = 1.70 ± 0.03

E(z)≡H(z)/Ho

The baryon acoustic oscillation peak A

$$A \equiv \sqrt{\Omega_{\rm m}^0} E(z_1)^{-1/3} \left[\frac{1}{z_1} \int_0^{z_1} \frac{dz'}{E(z')} \right]^{2/3}$$

where z_1 = 0.35

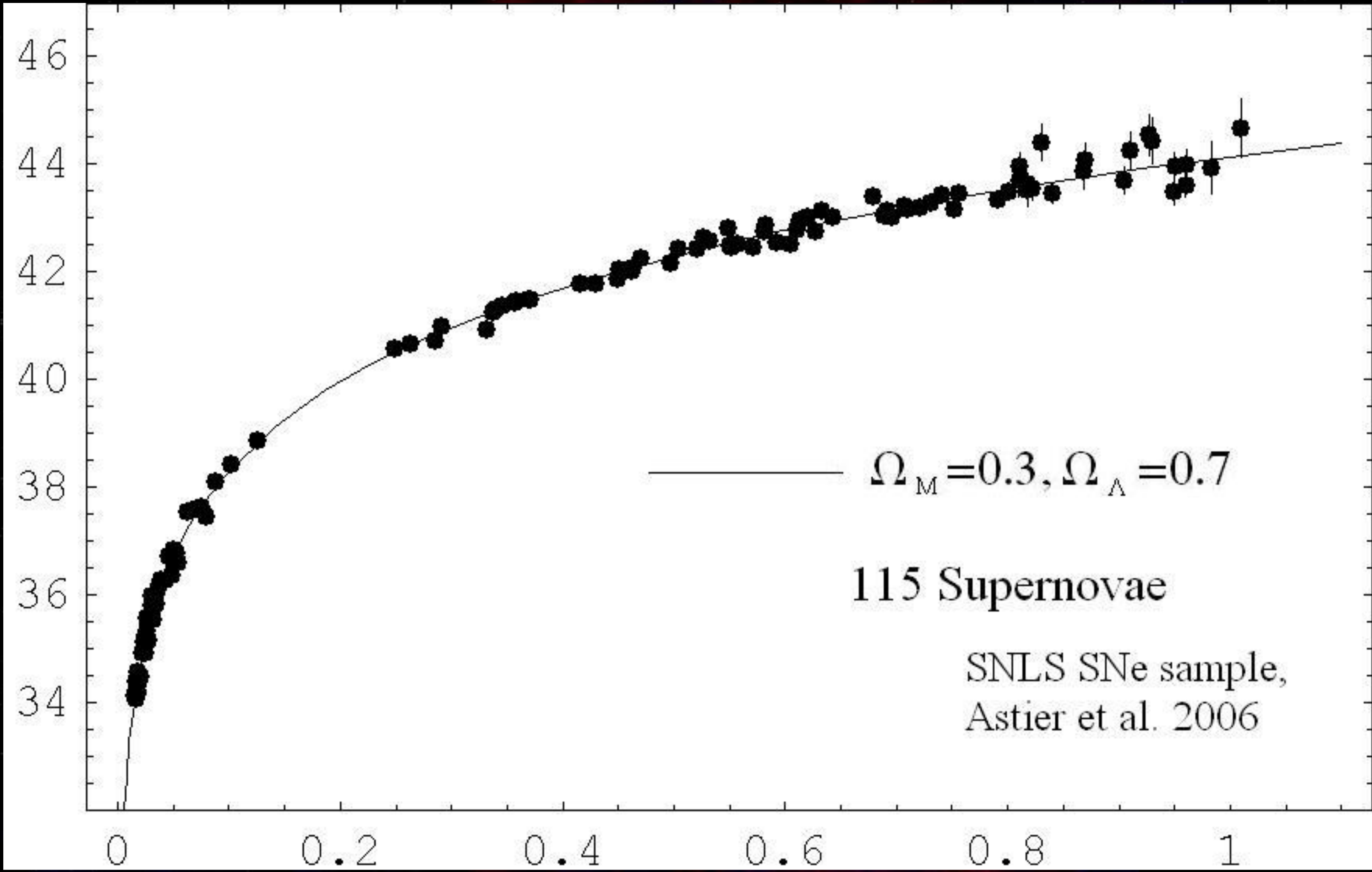
A_{obs} = 0.469 ± 0.017

$$\chi^2_{\rm CMB} = \left(\frac{R - R_{\rm obs}}{\sigma_R} \right)^2, \qquad \chi^2_{\rm LSS} = \left(\frac{A - A_{\rm obs}}{\sigma_A} \right)^2$$

Then, the joint χ2 function becomes

$$\chi^2 = \chi^2_{\rm SN} + \chi^2_{\rm CMB} + \chi^2_{\rm LSS}$$

Concordance Model, Open and Flat Models dominated by matter,



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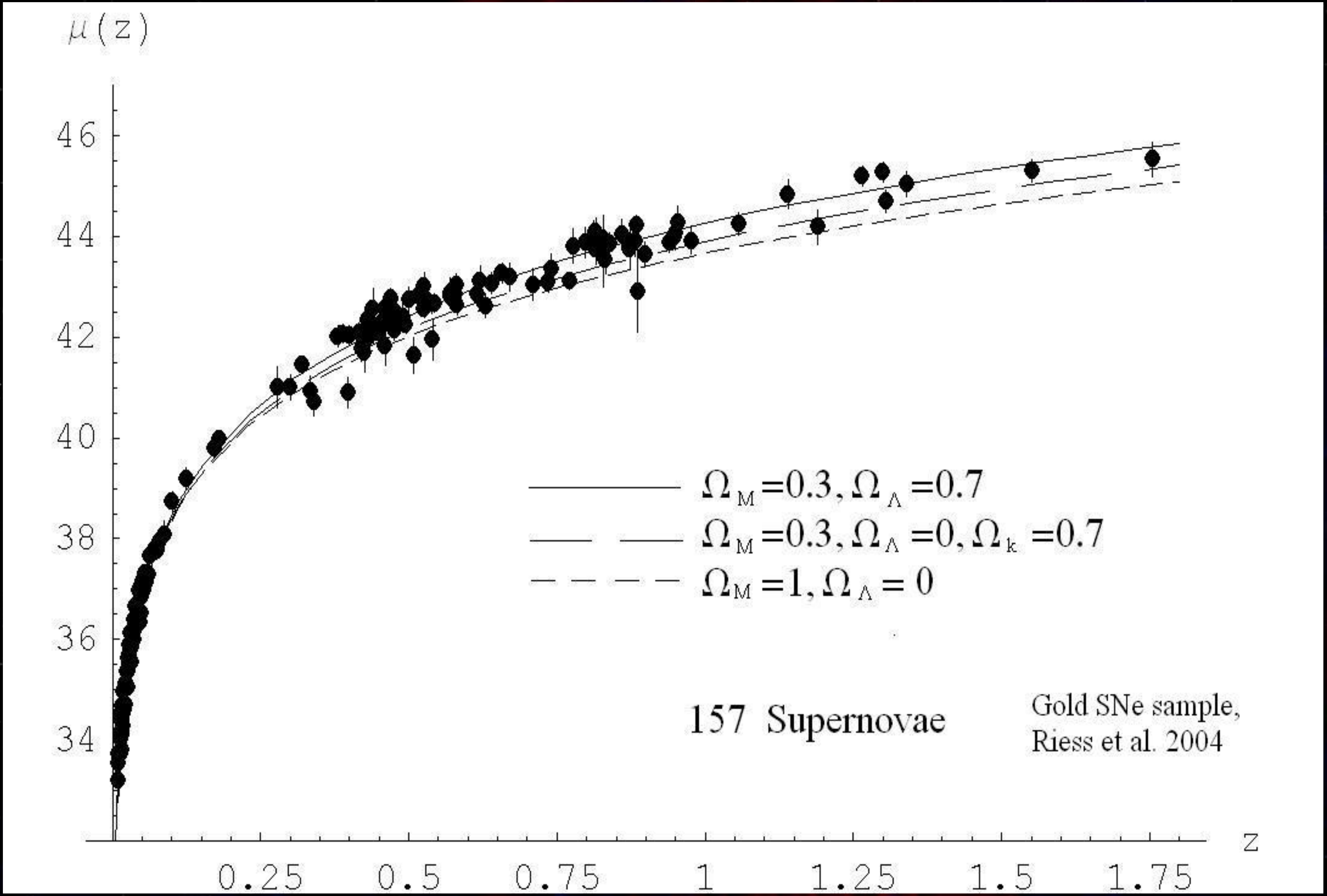

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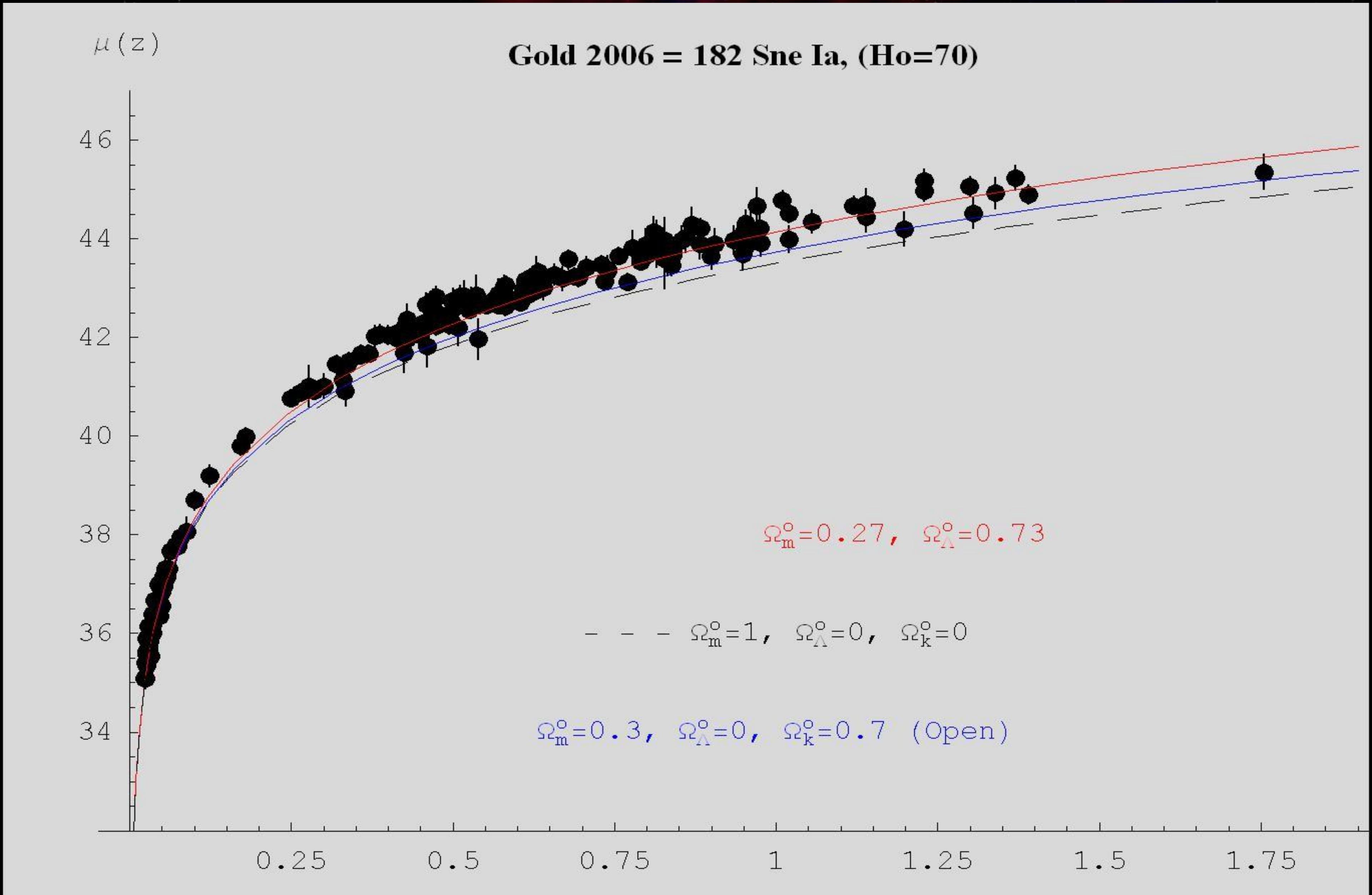
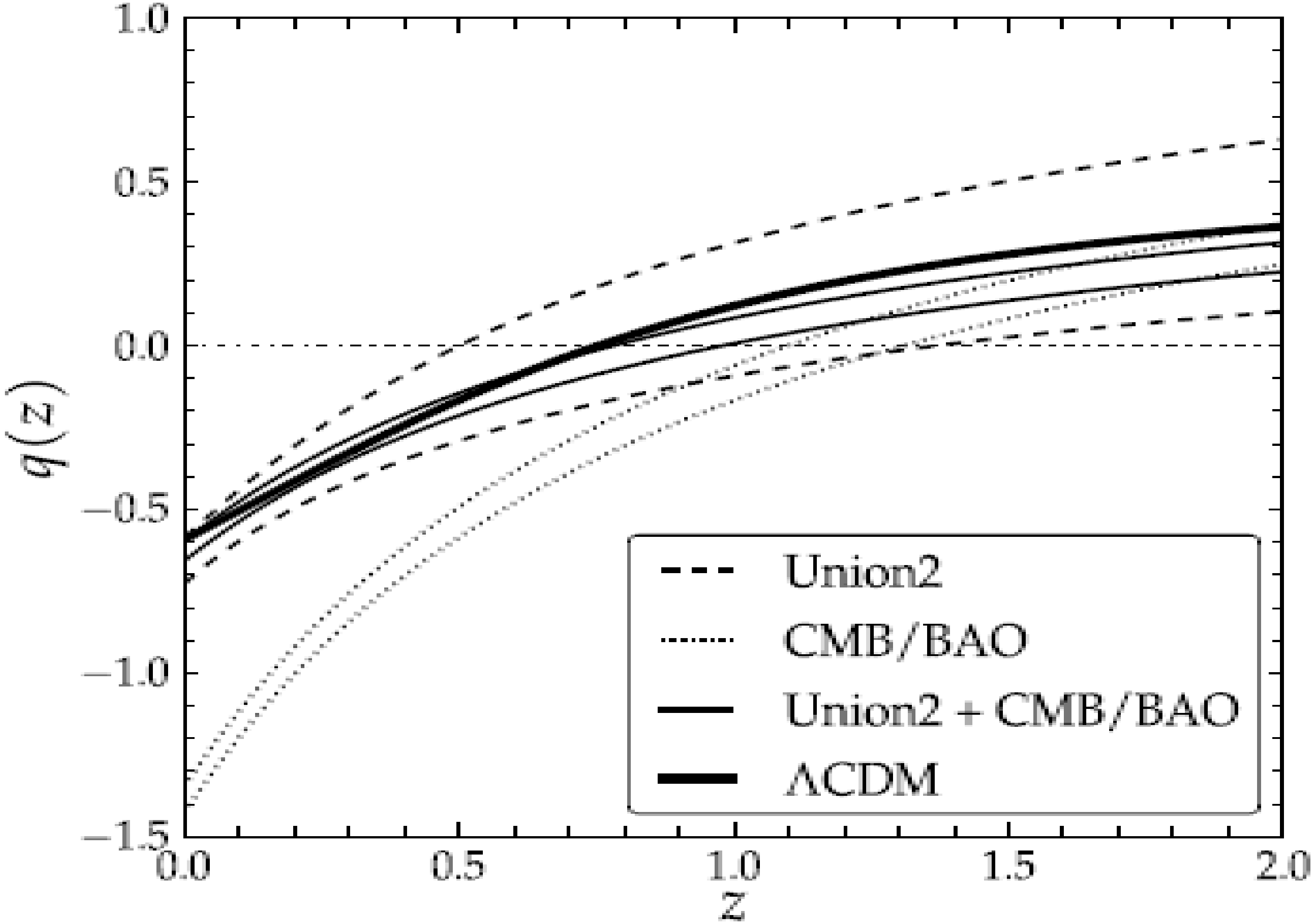


Constraints on Deceleration Parameter



5. - Kinematic Evidence for Aceleration: Lineal Anzatz.

Deceleration Parameter:

$$q(z) \equiv -\frac{a\,\ddot{a}}{\dot{a}^2}$$

Expanding in the lineal anzatz:

$$q(z) = q_0 + Q_0 z$$

The Hubble Parameter is:

$$H(z) = H_0 \exp\left[\int_0^z \{1 + q(u)\}\, d\,Ln(1+u)\right]$$

In a Flat universe k = 0, we have the luminosity distance:

$$d_L(z, H_0, q_0, Q_0) = \frac{c(1+z)}{H_0} \int_0^z \frac{du}{(1+u)^{1+q_o-Q_o}\, e^{u\,Q_o}}$$

4.- Estimación Bayesiana de of Parámetros.

$$X, Y, I = \text{Proposiciones.}$$

Regla producto para Probabilidades:

$$prob(X, Y \mid I) = prob(X \mid Y, I) \cdot prob(Y \mid I)$$

Pero Tenemos: $$prob(X, Y \mid I) = prob(Y, X \mid I)$$ $\Rightarrow$

Teorema de Bayes:

$$prob(X \mid Y, I) = \frac{prob(Y \mid X, I) \cdot prob(X \mid I)}{prob(Y \mid I)}$$

$$prob(X \mid Y, I) = \text{Densidad de probabilidad posterior}$$

$$prob(Y \mid X, I) = \text{Likelihood function.}$$

$$prob(X \mid I) = \text{Densidad de probabilidad Previa}$$

$Y = D =$ Muestra de datos de distancia modular de Sne Ia.

$X = \mathrm{H}_0, \Omega_i^0,$ etc. Modelo Cosmológico.

$I =$ Otras observaciones o suposiciones sobre: $\mathrm{H}_0, \Omega_i^0 = \Omega_\Lambda^0, \Omega_M^0$

El teorema de Bayes se escribe como:

$$\boxed{prob(X \mid D, I) \propto prob(D \mid X, I) \cdot prob(X \mid I)}$$

Asumimos: Densidad de probabilidad para el k-data:

$$\boxed{\mu_k \equiv \mu_k^{obs} \pm \sigma_k}$$

$$\boxed{prob\,(\mu_k^{obs} | X, I) \equiv \left(\frac{1}{\sigma_k \sqrt{2\pi}} \right) Exp\left[-\frac{[\mu_k^{teo}(z_k, X) - \mu_k^{obs}]^2}{2\sigma_k^2} \right]}$$

Tenemos la "Likelihood function" para *n datos* observados:

$$prob(D \mid X, I) \equiv \prod_{k=1}^{n} prob(\mu_k^{obs} \mid X, I) = \mathrm{A}\, Exp\left(-\frac{\chi^2}{2}\right)$$

Donde tenemos la distribución estadística Chi-cuadrada:

$$\chi^2(X) \equiv \sum_{k=1}^{n} \frac{[\mu_k^{teo}(z_k, X) - \mu_k^{obs}]^2}{\sigma_k^2}$$

Finalmente tenemos el Teorema de Bayes:

$$prob(X \mid D, I) \propto \mathrm{A}\, Exp\left(-\frac{\chi^2}{2}\right) \cdot prob(X \mid I)$$

Densidad de Probabilidad Posterior

"Likelihood density"

Densidad de Probabilidad previa.

Marginalización de parámetros.

Si el modelo *X* es parametrizado por (*n*+1) parametros:

$$X = (x_1, x_2, \ldots, x_{n+1})$$

Marginalizamos sobre uno de los parámetros:

$$x_{n+1} \in [a, b]$$

$$prob(\overline{X} \mid D, I) = \int_a^b prob(X \mid D, I)\, dx_{n+1} = \mathrm{A} \int_a^b Exp\left(-\frac{\chi^2}{2} \right) \cdot prob(X \mid I)\, dx_{n+1}$$

Ahora tenemos *n* parámetros:

$$\overline{X} = (x_1, x_2, \ldots, x_n)$$

Densidades de probabilidad (DP) previa más usados:

$$prob(X \mid I) = \delta(x_{n+1} - x^*_{n+1})$$

DP previa Delta de Dirac.

$$prob(X \mid I) = \text{constant}$$

DP previa Constante.

$$prob(X \mid I) = \exp\left[-\frac{(x_{n+1} - x^*_{n+1})^2}{2\sigma^2_{n+1}} \right]$$

DP previa Gaussiana.

Definición de una nueva distribución Chi-cuadrada:

$$\text{Prob}\,(\overline{X}\,|\,D,I) \equiv B\cdot Exp\left[-\frac{\tilde{\chi}^2(\overline{X})-\tilde{\chi}^2_{\min}}{2}\right]$$

Tenemos un máximo de Probabilidad en el mínimo de la distribución Chi-cuadrada:

$$\tilde{\chi}^2_{\min} = \tilde{\chi}^2(\overline{X}_{be})$$

$$\overline{X}_{be} = \text{mejor estimación para parámetros } (x_1, x_2, ..., x_n).$$

Regiones de confianza en los parámetros $\overline{X} = (x_1, x_2, ..., x_n)$ **se calculan usando:**

$$\tilde{\chi}^2(\overline{X}) - \tilde{\chi}^2_{\min} \equiv \Delta\tilde{\chi}^2$$

$\Delta\chi^2$ as a Function of Confidence Level and Degrees of Freedom

	ν					
p	1	2	3	4	5	6
68.3%	1.00	2.30	3.53	4.72	5.89	7.04
90%	2.71	4.61	6.25	7.78	9.24	10.6
95.4%	4.00	6.17	8.02	9.70	11.3	12.8
99%	6.63	9.21	11.3	13.3	15.1	16.8
99.73%	9.00	11.8	14.2	16.3	18.2	20.1
99.99%	15.1	18.4	21.1	23.5	25.7	27.8