Black Holes in the Water Tap

(Physical models dual to black holes)

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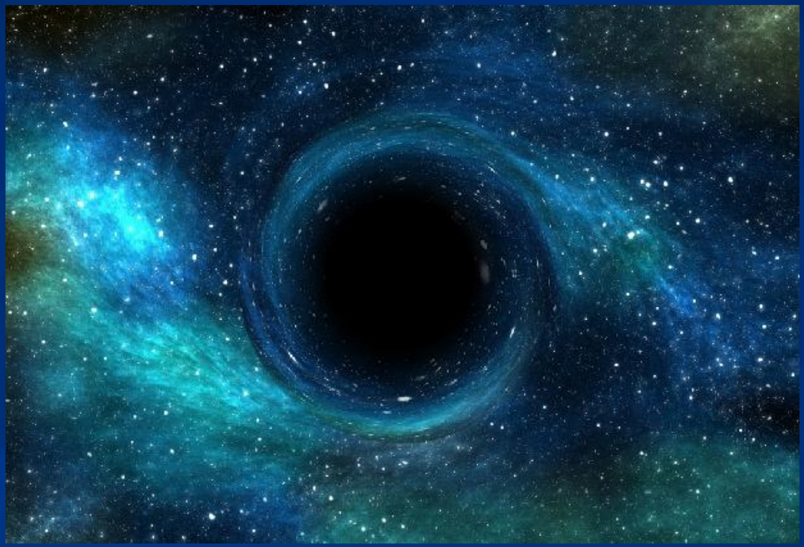
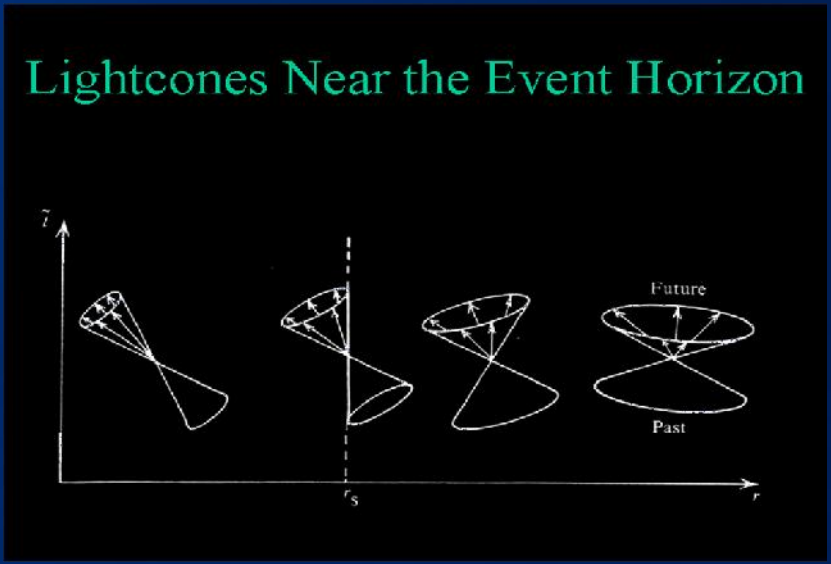
Black Holes



Region in space of incredibly high density

Light cone/Event horizon



Acoustic Black Holes

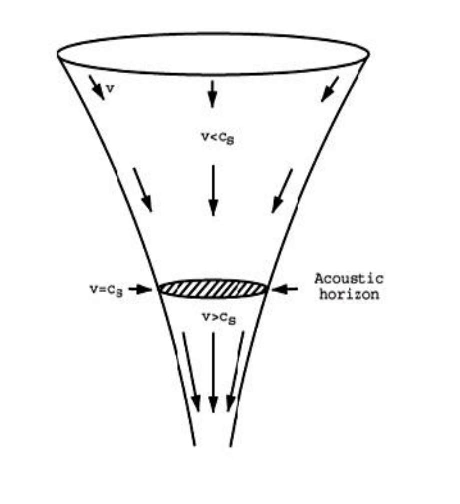
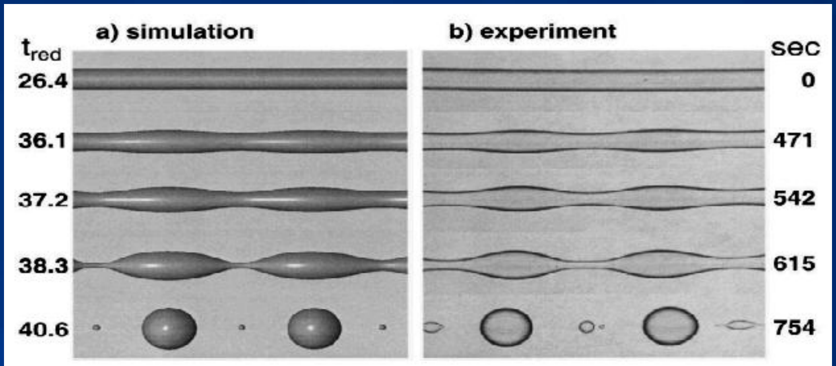


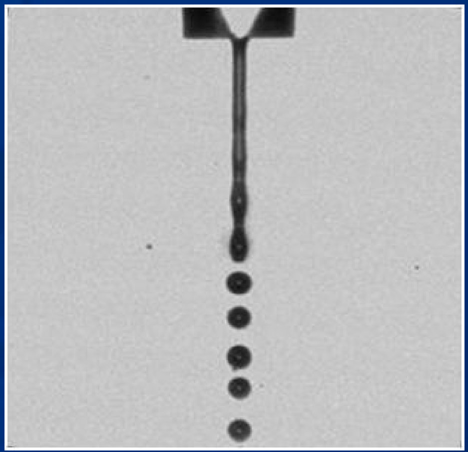
Figure 2. A toy model exhibiting an acoustic horizon. The arrows indicate the fluid flow velocity, with longer arrows for faster flow. The analogue event horizon (acoustic event horizon) occurs when the fluid velocity becomes equal to the sound velocity.

Causal Structure

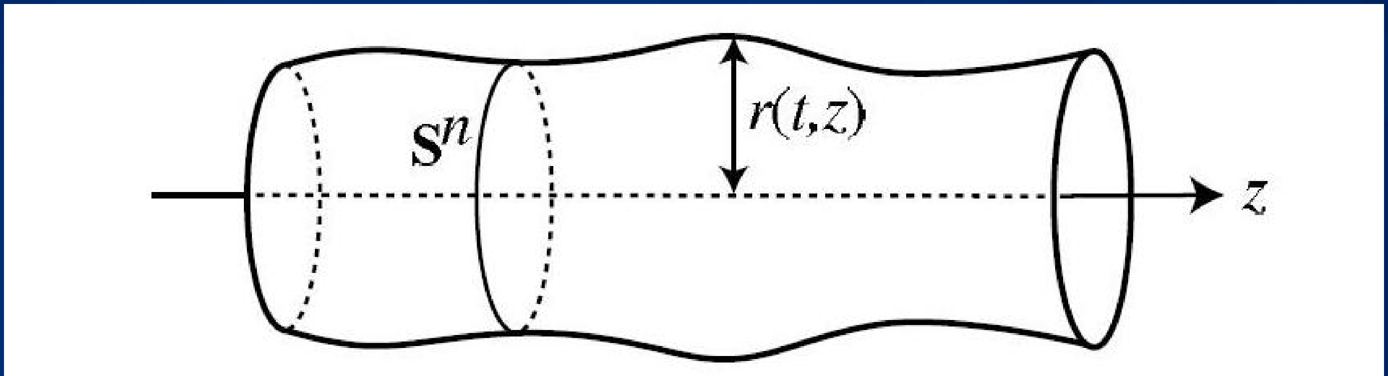
Real fluids



Plateau-Rayleigh
Instabilities



Black Holes in the Water Tap



Axially symmetric black hole

$$V(t) = \Omega_n \int_P dz\, r^{n+1}(z,t)$$

$$A(t) = (n+1)\Omega_n \int_P dz \sqrt{1+{r_z}^2}\, r^n(z,t)$$

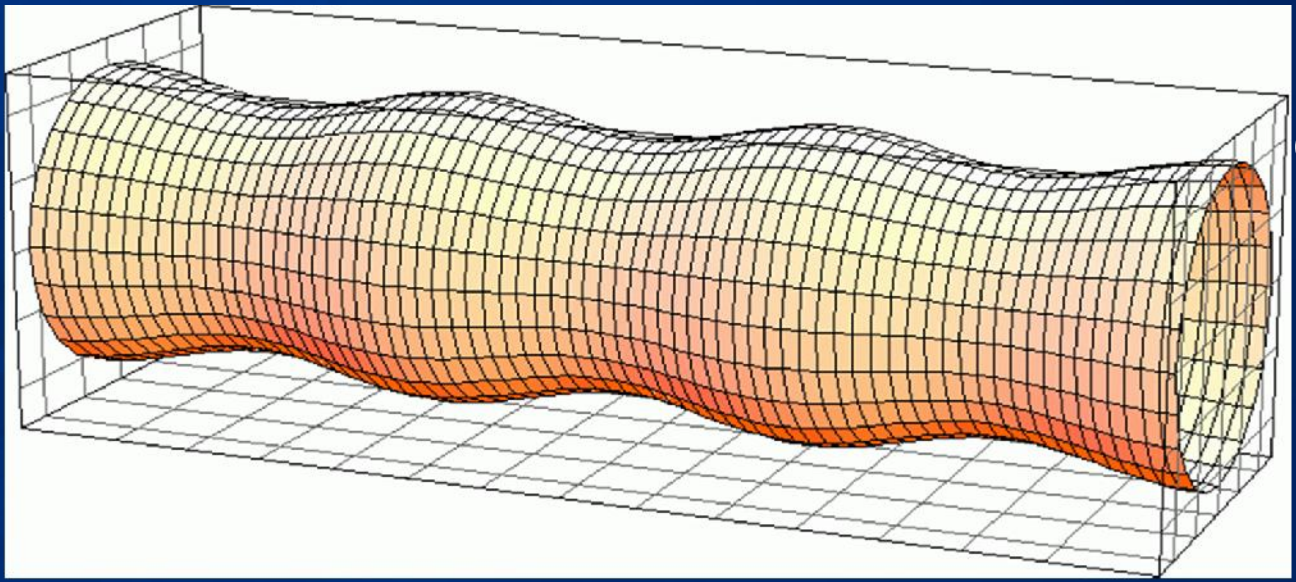
Fixed volume

Minimized area

Gregory-Laflamme Instabilities

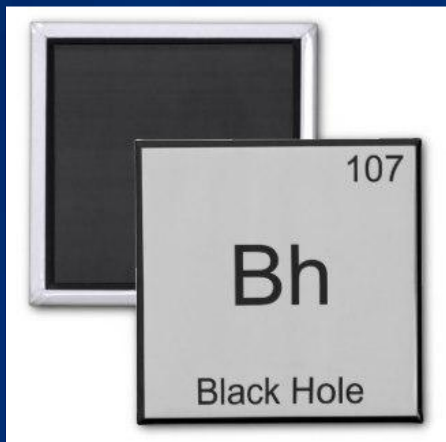
$$\partial_t r = \frac{B}{r^n}\partial_z\left[\frac{r^n}{\sqrt{1+{r_z}^2}}\partial_z\left(\frac{n}{r\sqrt{1+{r_z}^2}} - \frac{r_{zz}}{(1+{r_z}^2)^{3/2}}\right)\right]$$

4-th order differential equation



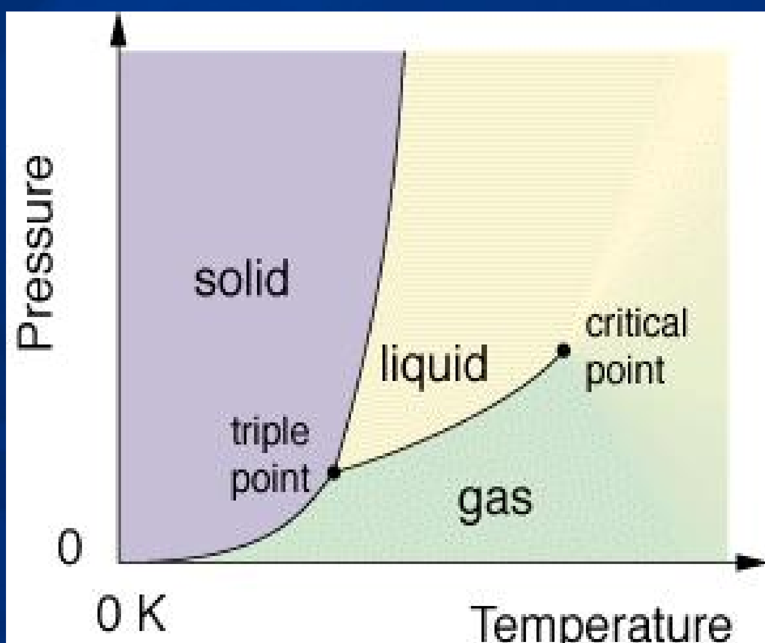
Gregory-Laflamme
Instabilities

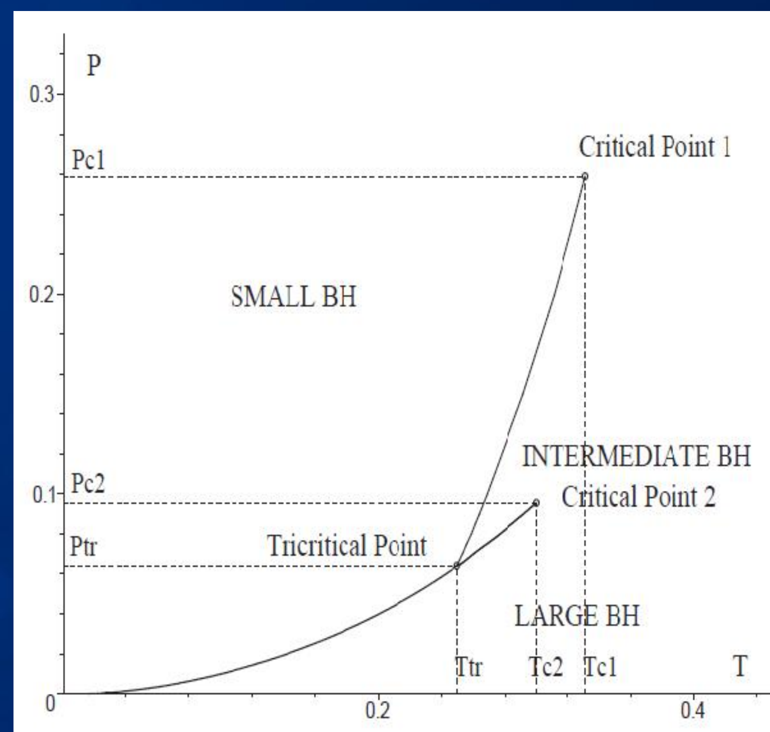
Black Hole Chemistry



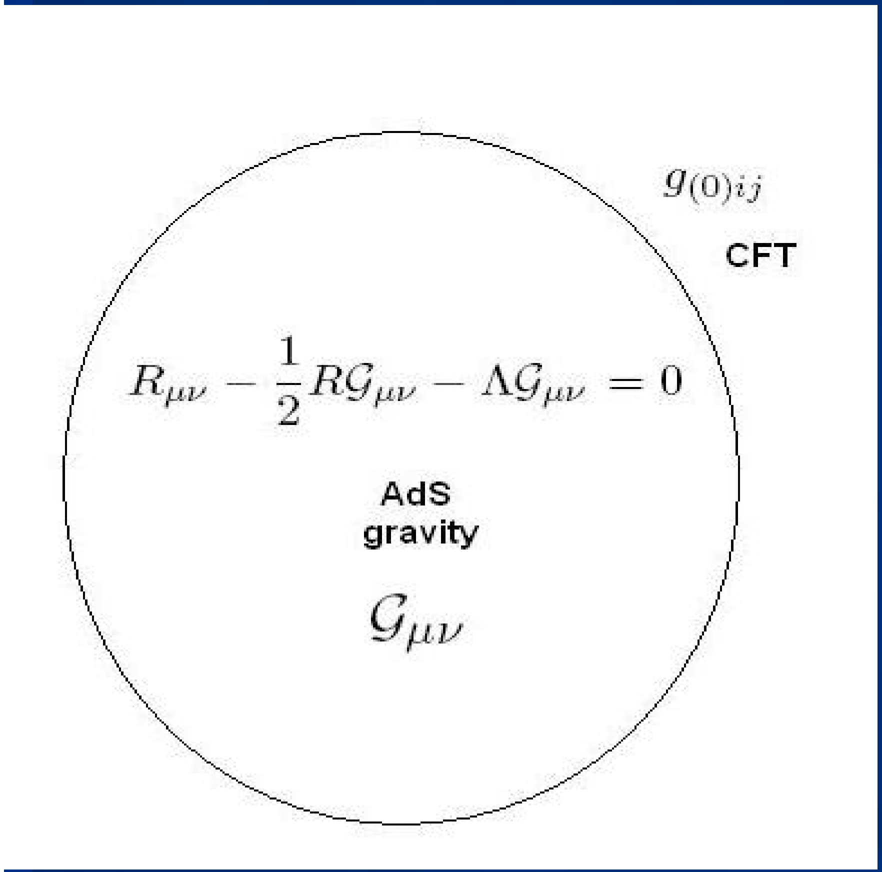
$$dH = TdS + VdP + \cdots \leftrightarrow dM = \frac{\kappa}{8\pi} dA + VdP + \cdots$$

First Law　　　　　　First Law





Gravity / Field Theory Duality



- Conjectured in 1998 (Maldacena)

- Duality: gravity theory in (d+1) dim/ boundary field theory

- *Holographic Principle* ('t Hooft and Susskind)

- Duality strong/weak coupling.

Applications of Gravity/Field Theory Duality

- Gravity/ QCD
  Dual models of quarks

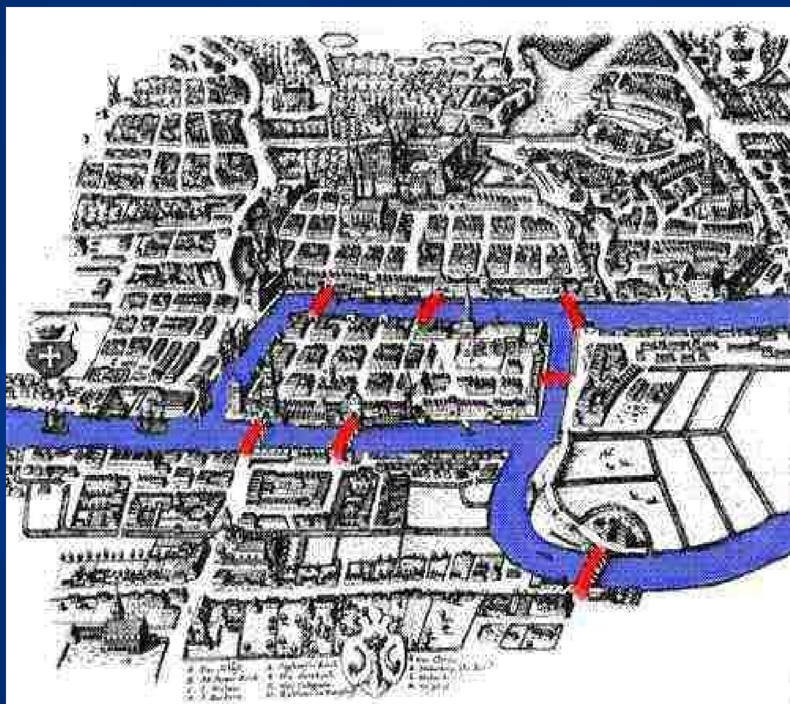
- Gravity / Hydrodynamics
  Quark-gluon plasma

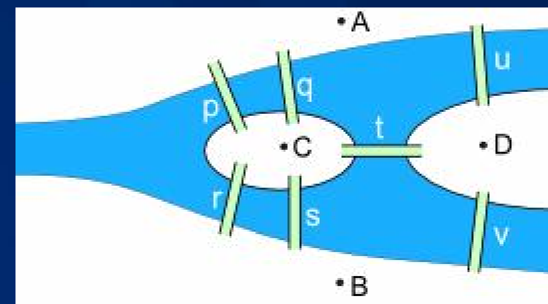
- Heavy Ion Collisions
  Relativistic Heavy Ion Collider (RHIC)

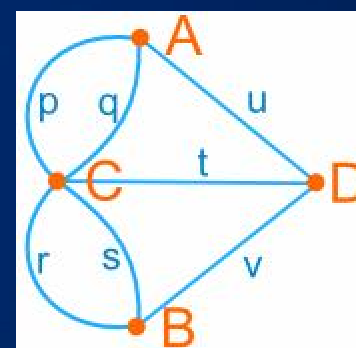
- Gravity / Condensed Matter
  Non-relativistic boundary theories

Topology

Seven Bridges of Königsberg (Euler, 1736)







Planar graphs, Topological Invariants,
Topological Phases of Matter (Nobel 2016),
etc.

Five-color theorem

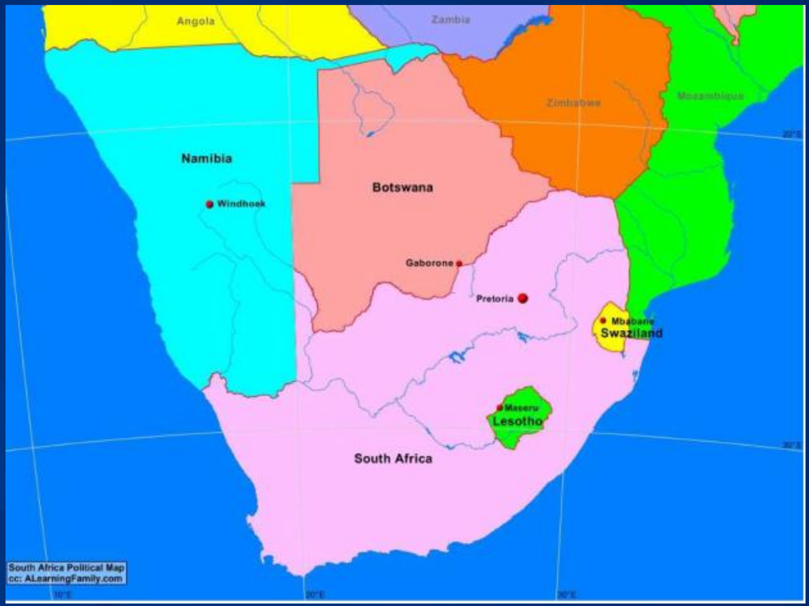
-Any political map can be colored -at most- with 5 colors

-Problem proposed by Guthrie in 1852.

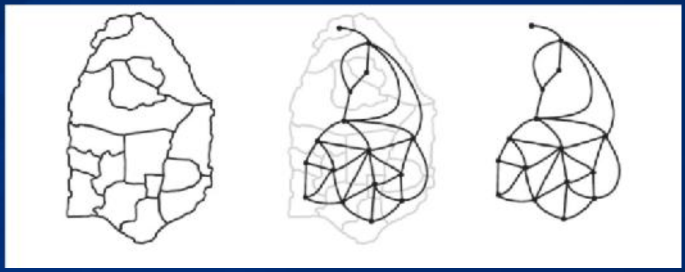
-Proof given by Heawood in 1890.

Five-color theorem

-Countries are simply-connected regions



-Any map is a planar graph: each region is a vertex and each edge is the border shared by two regions.



Five-color theorem

-One can always add edges such that every face is bounded by three edges. Then, $3F \leq 2E$

-From the Euler formula we have

$$F - E + V = 2$$

$$E \leq 3V - 6$$

-Every planar graph has vertices of degree five or less.

$$\chi = F - E + V$$

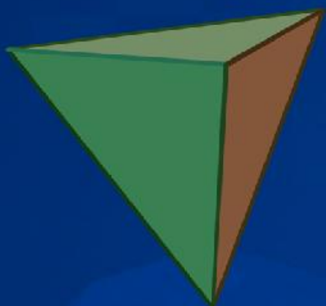
Euler characteristic

Topological Invariants (Euler characteristic)

Tetrahedron

F – E + V =

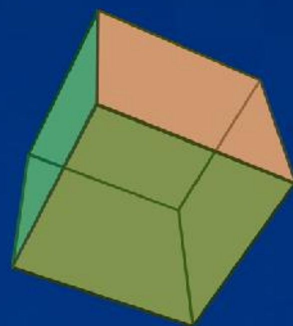
4 – 6 + 4 = 2



Hexahedron

F – E + V =

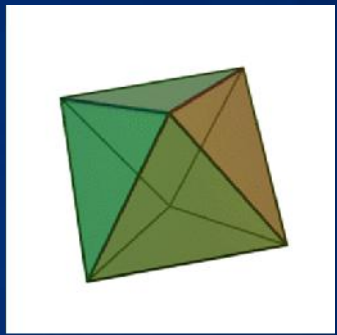
6 – 12 + 8 = 2



Octahedron

F – E + V =

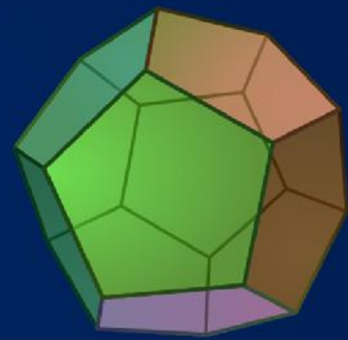
8 – 12 + 6 = 2



Dodecahedron

F – E + V =

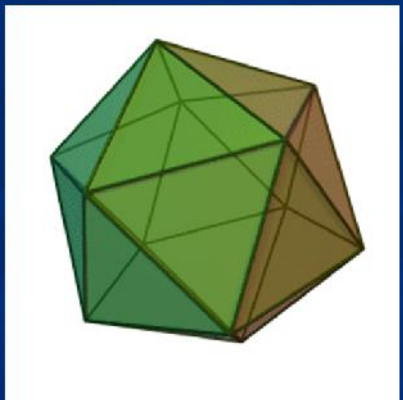
12 – 30 + 20 = 2



Icosahedron

F – E + V =

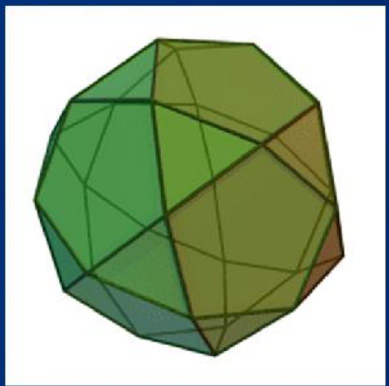
20 – 30 + 12 = 2



Icosidodecahedron

F – E + V =

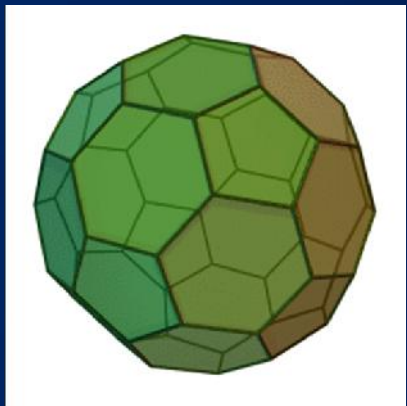
32 – 60 + 30 = 2



Truncated Icosahedron

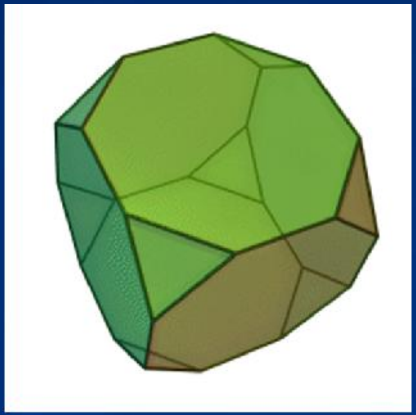
F – E + V =

32 – 90 + 60 = 2



Truncated Polyhedron

$\chi = 2$

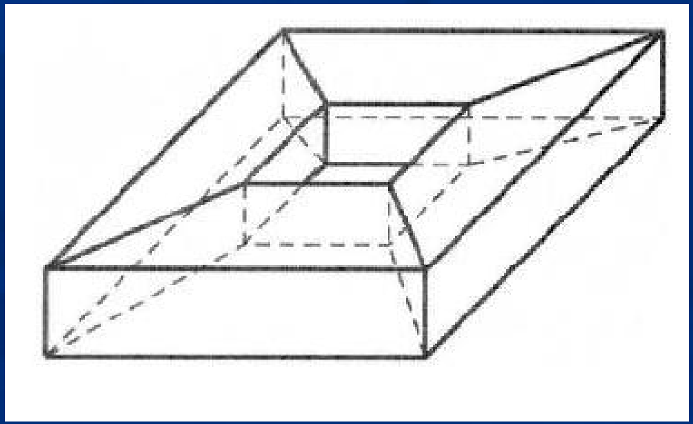


Continuum Limit (Sphere)

$\chi = 2$

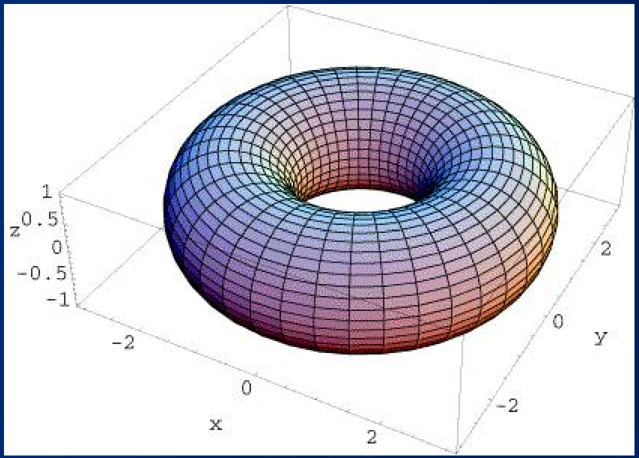
Polyhedron with one hole

$\chi = 16 - 48 + 32 = 0$



Continuum Limit (Torus)

$\chi = 0$

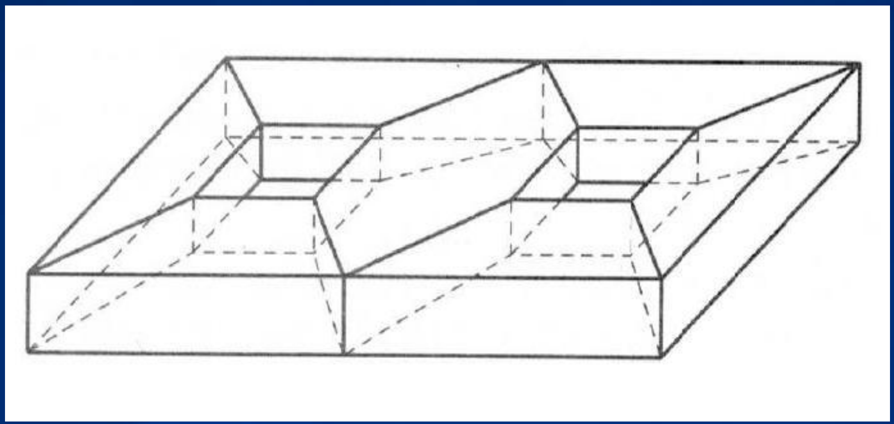


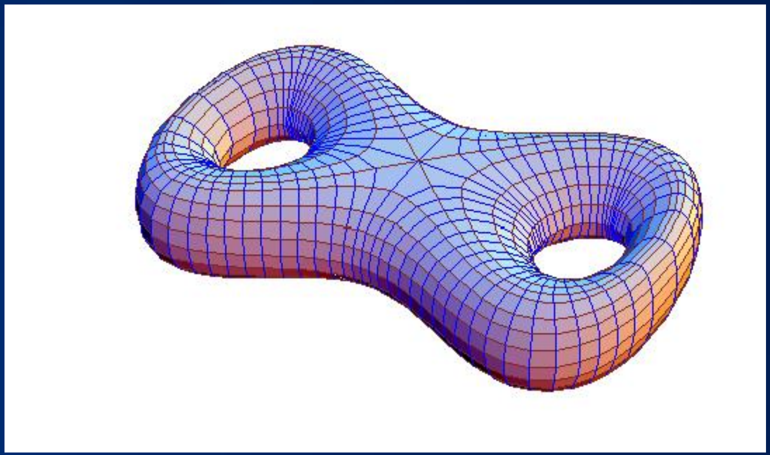
Polyhedron with two holes

$\chi = 30 - 96 + 64 = -2$

Continuum Limit (Double Torus)

$\chi = -2$





$$\chi = 2\,(1 - g)$$

g is the genus of the 2-dim manifold

Topological Invariants in Electrodynamics

$$\nabla \cdot \mathbf{E} = \rho$$

Gauss Law

$$-\frac{\partial}{\partial t}\mathbf{E} + \nabla \times \mathbf{B} = \mathbf{J}$$

Ampère’s Law

$$\partial_\mu F^{\mu\nu} = J^\nu$$

Maxwell Eq’s

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Field strength

4-vector potential

$$A_\mu = (-\phi, \mathbf{A})$$

4-vector current

$$J^\mu = (\rho, \mathbf{J})$$

$$E^i = F^{0i}$$

Electric field

$$B^i = \frac{1}{2}\epsilon^{ijk}F_{jk}$$

Magnetic field

$$\nabla \cdot \mathbf{B} = 0$$

Gauss Law for Magnetic Field

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$

Faraday's Law

$$\partial_\mu {}^*F^{\mu\nu} = 0,$$

Bianchi Identity

$$^*F^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Dual field-strength

$$I_{EM} = \int d^4x\,(E^2 - B^2) = \frac{1}{4}\int d^4x\,F_{\mu\nu}F^{\mu\nu} + \alpha \int d^4x F_{\mu\nu} * F^{\mu\nu}$$

Coupling constant α remains arbitrary

Topological Invariants and AdS Gravity

$$I = \frac{1}{16\pi G} \int_M d^4x \sqrt{-\mathcal{G}} (R - 2\Lambda) + \alpha \int_M \mathcal{E}_4$$

EH + Gauss-Bonnet

Remarks:

- Gauss-Bonnet term does not modify the equations of motion.
- Bulk dynamics cannot determine GB coupling only by itself.

Conserved quantities

$$Q(\xi) = K(\xi) + q_{GB}(\xi)$$

$$K(\partial_t) = \frac{M}{2} + \lim_{r\to\infty} \frac{r^3}{2\ell^2}$$

$$q_{GB}(\partial_t) = \frac{4\alpha}{\ell^2} \left(\frac{M}{2} - \lim_{r\to\infty} \frac{r^3}{2\ell^2} \right)$$

R.Aros et al, Phys. Rev. Lett. 84, 1647 (2000)

- Black Hole Thermodynamics

$$G = TI_E = -TS + \frac{M}{2}\left(1 + \frac{4\alpha}{\ell^2}\right) - \frac{\pi r^3}{4G\ell^2}\left(1 - \frac{4\alpha}{\ell^2}\right)$$

Correct Black Hole Thermodynamics is obtained only for $\alpha = \frac{\ell^2}{4}$

R.O., J. High Energy Phys. 0506, 023 (2005)

Moral: Topological Invariants are essential for (real) life

Topological invariants, Black Holes and Relativistic Hydrodynamics

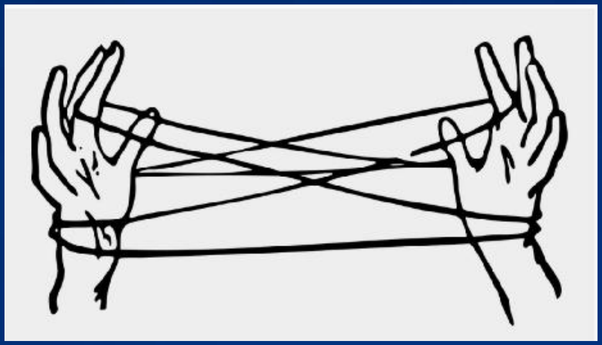
Holographic Stress Tensor/ Cotton Tensor Duality $\leftrightarrow$ Addition of Top. Invariants (Gauss-Bonnet + Pontryagin)

O. Miskovic and R.O., PRD**79**: 124020 (2009).

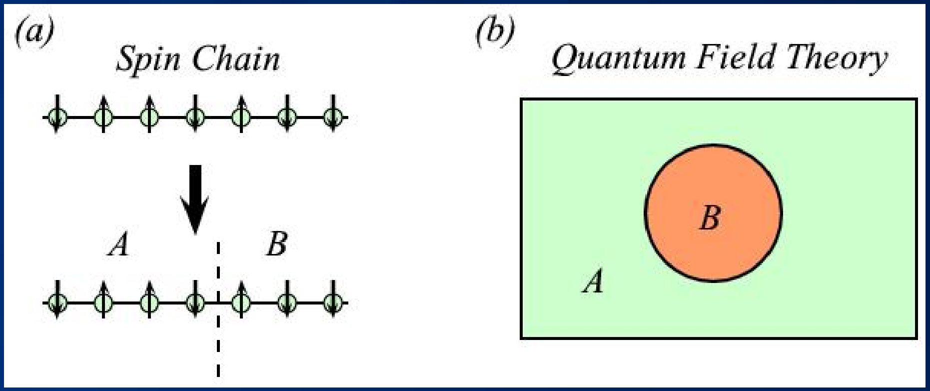
R.Araneda, R.Aros, O. Miskovic and R.O., PRD**93**: 084022 (2016).

Bonus: Formulation of Gravity à la Electromagnetism/Yang-Mills

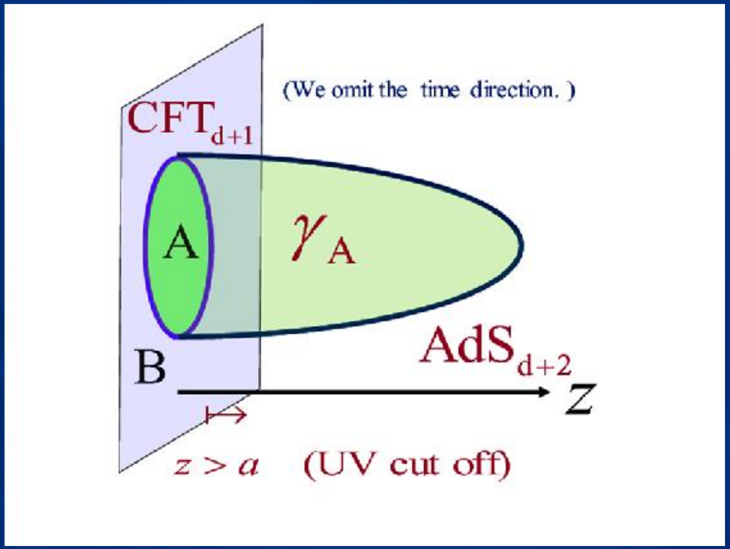
Topological invariants and Entanglement Entropy



Quantum Entanglement



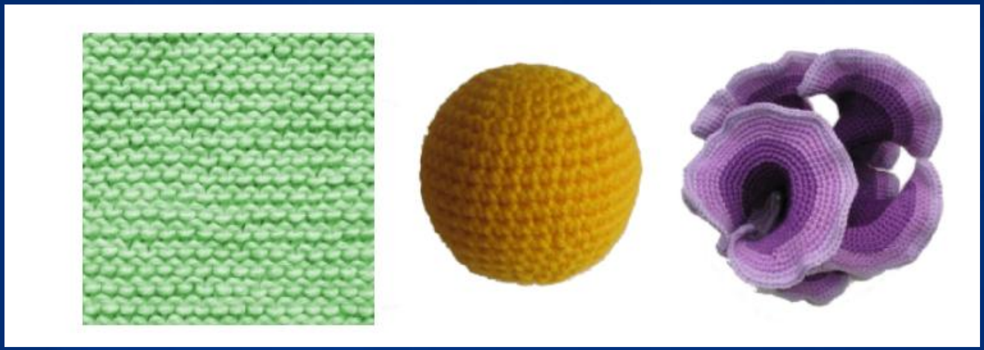
Sub-system Entanglement

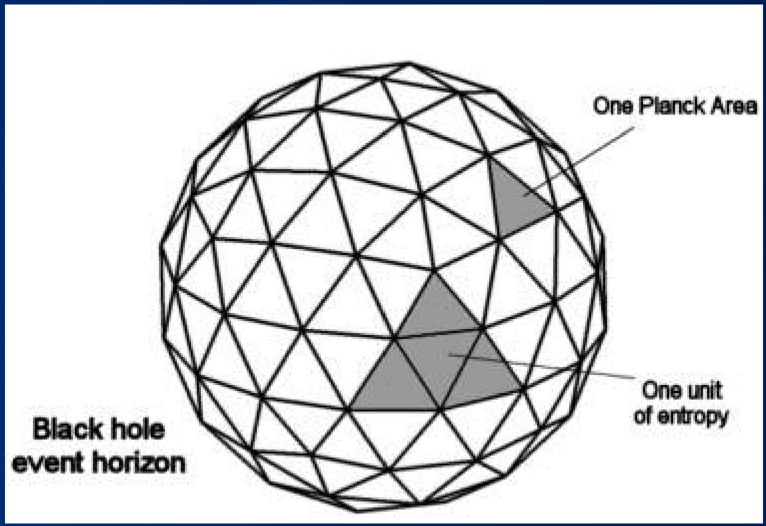


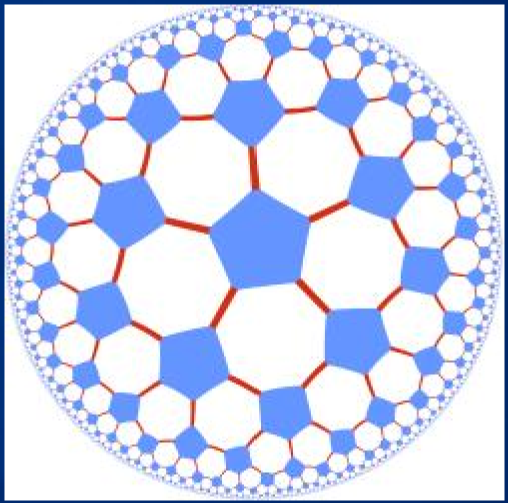
Holographic EE: Ryu-Takayanagi '06

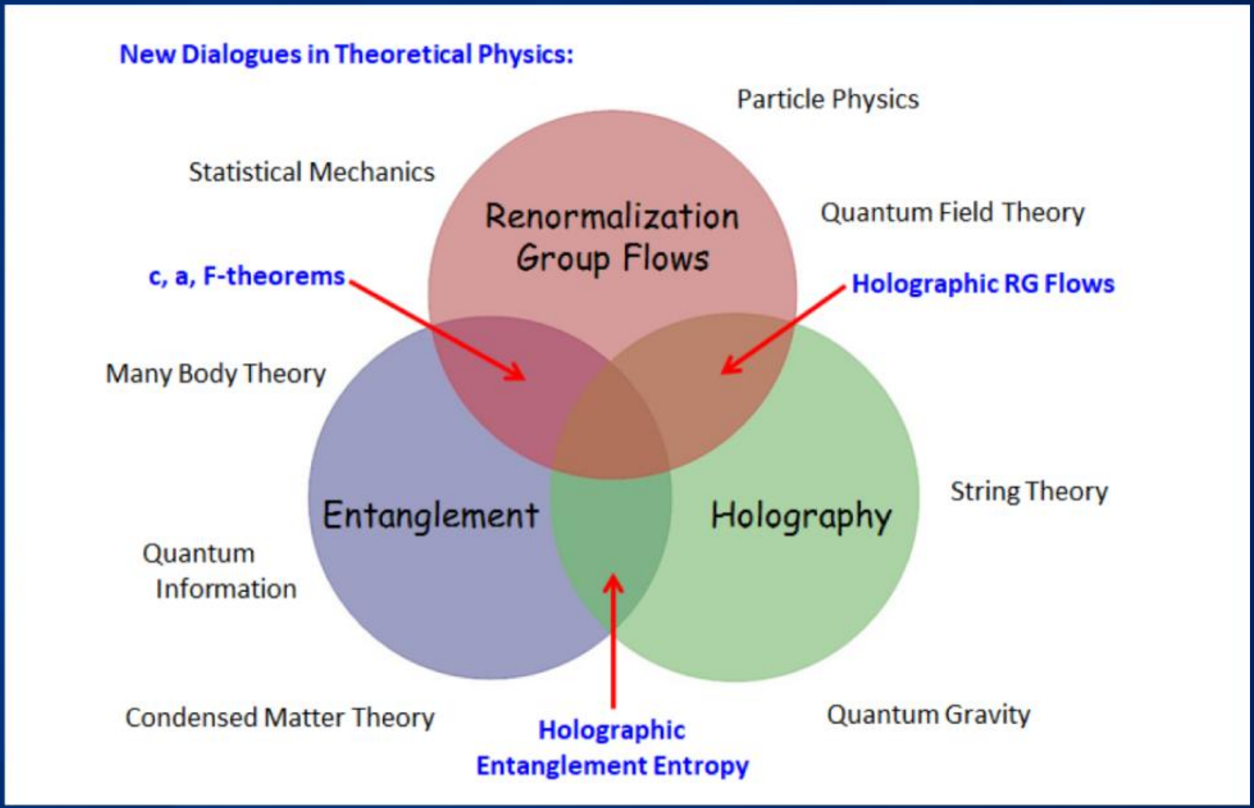
G.Anastasiou, I.J. Araya and R.O., PRD 97, 106011 (2018)
G.Anastasiou, I.J. Araya and R.O., PRD 97, 106015 (2018)
G.Anastasiou, I.J. Araya, C.Arias and R.O., JHEP 1808, 136 (2018)

Tensor Networks and Quantum Information Theory








New Dialogues in Theoretical Physics:
Particle Physics
Statistical Mechanics
Renormalization Group Flows
Quantum Field Theory
c, a, F-theorems
Holographic RG Flows
Many Body Theory
Entanglement
Holography
String Theory
Quantum Information
Condensed Matter Theory
Holographic Entanglement Entropy
Quantum Gravity

Viva la Relativity!!!